How to Fix a Coordination Failure: A “Super-Pigouvian” Approach

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A central concern in industrial policy discussions is that sector-specific external economies of scale may create multiple equilibria—and therefore the potential for coordination failure. Unfortunately, Pigouvian policies that address market failures on the margin do not remove the risk of mis-coordination globally. I propose a new “super-Pigouvian” (SP) policy that retains the decentralized spirit of Pigouvian policy—regulating prices rather than quantities—but also prevents coordination failure. The main idea behind SP is to subsidize market behavior, both on and off the equilibrium path, according to the population’s willingness to pay for the externalities that (a) those behaviors generate directly, like Pigou, and also (b) they generate indirectly by affecting other households’ choices. After demonstrating SP’s welfare properties theoretically, I quantify them in a dynamic model of structural transformation calibrated to South Korea’s heavy and chemical industry drive in the 1970s. SP modestly improves welfare compared to the worst equilibrium under Pigouvian policy.

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1 Introduction

Industrial policy is back in fashion. Even relatively free-market countries like the United States are promoting investment in geographically concentrated “innovation zones,” subsidizing R&D in the strategic semiconductor industry, and backing nascent green technologies in the hopes they become profitable.

The standard economic rationale for industrial policy is to correct market failures introduced by externalities among firms or between firms and households. These market failures come in two distinct varieties. First, externalities cause individual economic actors to under- or over-prioritize actions relative to their marginal social value. For example, firms may be unwilling to invest in a green production method if they bear the full cost while others share in the benefit of lower pollution. Second, externalities can cause miscoordination between economic actors, leading to coordination failures that are impossible with complete markets. For example, suppose that green technologies become cheaper the more firms adopt them. Even if it would be efficient for all firms to choose the green technology, each individual firm may choose not if no one else does, either.

The dominant industrial policy paradigm for correcting these externalities is Pigouvian taxation and subsidization. Intuitively, a Pigouian policymaker taxes or subsidizes an activity—for example, a firm’s investment in green technology—until its private marginal cost equals its social marginal cost [Pigou, 1924]. This same logic underlies policies as wide-ranging as carbon taxes, R&D subsidies, and federal grants for basic science research [Nordhaus, 2006, Görg and Strobl, 2007, Mandt et al., 2020]. Even non-Pigouvian policies, like infant industry protection, are often justified as second-best versions of the Pigouvian ideal [Harrison and Rodríguez-Clare, 2010].

Despite its popularity with academics and policymakers alike, Pigouvian policy has a key limitation: it does not solve coordination failures. To see why, consider a simple example:

Workers can either work in the traditional sector, in which they have exogenous productivity, or the industrial sector, where their productivity increases in the number of other industrial workers—for instance due to knowledge spillovers. Even if full industrialization creates more output, Pigouvian policy can support the allocation where all workers choose the traditional sector. Intuitively, given that no one works in the industrial sector, no one stands to benefit from the increase in industrial productivity caused by any one workers decision to move to that sector. So the Pigouvian tax / subsidy is zero and there is no industrialization as long as—when there are no workers are in industry—wages are lower in industry than in the traditional sector.
In this paper, I propose a new, “super-Pigouvian” policy that retains the spirit of Pigouvian policy while also preventing coordination failure. Like Pigouvian policy, this new policy taxes and subsidizes private actions in order to align households’ private incentives with the social welfare. Moreover it retains the appealing decentralization of Pigouvian policy, augmenting prices rather than regulating quantities.

However, super-Pigouvian policy differs from Pigouvian policy in two essential ways. First, although both policies ensure households are paid for the full welfare impact of their actions, inclusive of externalities, super-Pigouvian taxation measures this impact in a more holistic way. Concretely, Pigouvian policy measures the impact of a household on welfare while holding fixed the behavior of all other households. Super-Pigouvian policy instead compensates each household for both these direct effects and the indirect effects of its actions on welfare through their influence on other households’ behavior.

The second essential aspect of super-Pigouvian policy helps to address a challenge posed by the first: How does any one household’s behavior affect another’s? So that this notion is well defined, I work in an alternative version of the traditional general equilibrium setup wherein (a) households act sequentially and (b) the economy must follow a competitive equilibrium starting from any history of past behavior. Within this setting, I require that Super-Pigouvian policy be set according to the same rule, regardless of past behavior. This condition—the second key aspect of its definition—shapes behavior off of the equilibrium path and therefore disciplines the effects of one household’s action on another’s. This differs from Pigouvian taxation, which need only be specified along an equilibrium path.

My main theoretical result is a first welfare theorem for super-Pigouvian policy. Namely, I show that any equilibrium in which policy is set according to the super-Pigouvian rule is Pareto efficient. Three ideas underlie this result. First, I study a dynamic model in which households act sequentially rather than simultaneously. This narrows the scope of coordination failure by allowing each actor to observe actions that have come before them, rather than guessing what they might be. Second, my approach compensates households for their welfare impacts accounting for effects on other, future behavior, which ensures that any given household behaves the same way a social planner would at any given moment. Third, the fact that super-Pigouvian policy is set off the equilibrium path ensures that a household who deviates away from an inefficient equilibrium knows that those who act afterwards will continue the efficient transition it began. The combination of these three factors implies efficiency by a logic akin to the principle of optimality in dynamic programming.

The idea that an alternative policy can improve on Pigouvian taxation may appear to conflict with standard results in public finance. As is well known, any set of taxes or subsidies that implements an efficient allocation must, at least locally to this allocation,
be Pigouvian. Consistent with this fact, I show that, in any strategy profile that can be implemented under super-Pigouvian policy—and so, according to my first welfare theorem, is efficient—taxes and subsidies are Pigouvian local to the equilibrium path. Where super-Pigouvian taxes and subsidies differ from those prescribed by a Pigouvian approach is in inefficient strategy profiles. There, the super-Pigouvian taxes and subsidies guarantee that the profile cannot be individually rational for some market participant and so is not an equilibrium. I moreover confirm that super-Pigouvian policy is not overly selective: as with standard Pigouvian policy, any efficient allocation can be implemented in a super-Pigouvian equilibrium (a second welfare theorem).

I also address the concern that super-Pigouvian policy may require more information on the behalf of a social planner. One of the advantages of standard Pigouvian policy is that it can be implemented using only information on the extent of externalities on the margin of the equilibrium path—or equivalently, households’ willingness to pay (WTP) for one another’s actions, holding fixed all future behavior. I show that, analogously to the Pigouvian case, a planner can verify whether policies are super-Pigouvian using only household WTPs for one another’s actions, provided that these WTPs account for actions indirect effects through their influence on future behavior. Importantly, each household can form these WTPs using only information about prices and transfers, without any knowing the efficient allocation.

After establishing these welfare properties in a general environment, I quantify the welfare gains from super-Pigouvian policy in a dynamic model of structural transformation that I calibrate using South Korea’s heavy and chemical industry (HCI) drive in the 1970s. This model captures the idea that there may be strong complementsaries between firms—for example steel manufacturers and industrial machine producers—who use one another’s output as an input to their own production [Rodrik, 1996, Buera et al., 2021]. When these complementarities are strong enough, entrepreneurs’ optimism or pessimism about industrialization can be self-fulfilling.

I take this model to the data using estimates of the HCI drive’s effects on firm entry from Lane [2022] and use the calibrated model to compute welfare under a range of policies. I find that Pigouvian taxation can support multiple equilibria at different levels of self-fulfilling optimism about entry into the industrial sector. Relative to the worst Pigouvian equilibrium, super-Pigouvian policy generates modest welfare gains equivalent to a ~2% increase in HCI value added. I also examine the mechanism by which super-Pigouvian policy incentivizes households to “break out” of inefficient Pigouvian equilibria. Super-Pigouvian policy offers large incentives for firm entry concentrated in regions of the parameter space where firms are nearly indifferent to entry.


**Literature:** This paper is not the first to propose a policy that can resolve coordination failures without requiring a social planner to know the efficient equilibrium path. Notably, Sandholm [2002, 2005, 2007] studies whether a social planner may implement the efficient allocation by simply committing to engage in Pigouvian taxation at any history the economy may reach. He shows that when—rather than acting with perfect foresight—households update their behavior myopically and with a small amount of noise, only the efficient strategy profile is played in a non-negligible fraction of periods in the long run. Fujishima [2013] provides a similar result in the case of perfect foresight, showing that in the limit of small frictions in updating behavior, (a) there always exists an equilibrium path from the current state to the efficient steady state and (b) starting from sufficiently close to that state, the only equilibrium converges to it.

Like the policy these papers propose, super-Pigouvian policy requires taxes to be set appropriately both on and off the equilibrium path. However, super-Pigouvian policy sets these taxes in a non-Pigouvian fashion. I show that this difference leads to stronger welfare properties: super-Pigouvian policy ensures efficiency along the entire path, not just at a steady state, and its efficiency properties do not rely on agents acting myopically or with vanishing frictions. More methodologically, I contribute to the public finance literature by proving the efficiency of a policy equilibrium using the principle of optimality from dynamic programming rather than first-order conditions combined with a convexity assumption. This approach may be useful for other analyses of policy design in non-convex economies.

Also related is a literature on dynamic mechanism design. One way to understand the welfare properties of super-Pigouvian policy is by analogy to a dynamic game of common interest with staggered actions. Super-Pigouvian policy ensures that households (a) act in the common interest and (b) even though they are price takers, act as if they can influence the equilibrium path. My first welfare theorem is then analogous to the efficiency of subgame perfect equilibria in sequential-action, discounted, dynamic games of common interest [Lagunoff and Matsui, 1997]. The super-Pigouvian idea that households should be compensated for their full contribution to welfare is also familiar from the well-known mechanism of Vickrey [1961], Clarke [1971], Groves [1973] and the more recently-proposed dynamic pivot mechanism [Bergemann and Välimäki, 2010]. The essential distinction between the approach of these papers and my own is that their mechanisms require households to share all of their private information with a central planner; this makes it easy to avoid coordination failure, as there is a single decision-maker. By contrast, I emphasize how super-Pigouvian policy

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1Methodologically, the one-shot deviation principle in discounted dynamic dynamic games is closely tied to the dynamic programming logic on which my proof relies.

2I also differ from this literature—as well as the one on subgame perfect implementation more generally, e.g. Moore and Repullo [1988]—by implicitly assuming that a social planner can elicit household willingnesses
can prevent coordination failure without a central planner needing to know the efficient allocation.

While the idea of coordination failure applies to many topics, it has received particular attention in the industrial policy sphere [Rosenstein-Rodan, 1943, Murphy et al., 1989, Ciccone, 2002, Rodrik, 2004]. From a modelling perspective, my main contribution to this literature is to nest richer static models in the spirit of Rodrik [1996], Rodriguez-Clare [1996] within the dynamic setup of Frankel and Pauzner [2000], Frankel et al. [2005]. On the estimation side, I calibrate my model using empirical moments from actual policy variation, rather than steady-state sufficient statistics as in Buera et al. [2021]. Finally, I assess the gains from a novel industrial policy.

Although I focus on industrial policy, my theoretical results apply to numerous other settings where positive externalities can create multiple equilibria. One important case is economic geography, where a long literature has considered whether local spillovers may lead to agglomeration in inefficient locations [Krugman, 1991, Davis and Weinstein, 2002, Bleakley and Lin, 2012, Allen and Donaldson, 2020]. Another application is to directed technological change, where the fact that successive innovations “build on the shoulders” of earlier inventions can create multiple equilibria [Acemoglu et al., 2012]. The theory also applies to business cycle models, in which a recent trend has been to rule out multiple “sunspot” equilibria with modelling assumptions, using tools from the global games literature [Angeletos and Lian, 2016]. In each of these contexts, many authors make modelling assumptions that rule out multiplicity or, if they allow for multiplicity, study policies that are necessary for efficiency but not typically sufficient [Guimaraes and Machado, 2018, Schaal and Taschereau-Dumouchel, 2015, Fajgelbaum and Gaubert, 2020]. Super-Pigouvian taxation is a policy that one can propose while neither foreclosing multiplicity through modelling choices nor narrowing the definition of policy success.

Outline: The paper is organized as follows. In Section 2, I provide a motivating example that introduces the main ideas of the paper in a simplified setting. Section 3 presents the general model. Sections 4 and 5 characterize the welfare properties of Pigouvian and super-Pigouvian properties, respectively, as well as the information required for a planner to implement them. Section 5.4 covers several extensions of the baseline model. Finally Section 6 quantifies the impacts of super-Pigouvian policy in a specialized model of industrialization, calibrated using the HCI drive in 1970s South Korea. Section 7 concludes.

to pay. Here I am motivated by the application to large economies in which any one agent’s preferences have a negligible effect on policy.
Figure 1: Timing and output in a simple model of industrialization. The structure of the tree represents the timing of household sectoral choice. \( T \) is the traditional sector and \( I \) is the industrial sector. Terminal nodes show the value of aggregate output at each allocation of households to sectors.

2 Motivating example

Before jumping into the formal model, I introduce the main ideas behind super-Pigouvian policy in an accessible example.

2.1 A simple model of industrialization

Two households \( i = 1, 2 \) sequentially decide whether to work in the traditional sector or the industrial sector. After both have chosen a sector, they each inelastically supply one unit of labor and production occurs.

The traditional and industrial sectors are competitive and produce a common final good. In the traditional sector, each household has productivity \( A > 0 \). The industrial sector features a productivity externality, with each household having productivity \( (L^I)^\alpha \), where \( L^I \) is the number of households in the industrial sector and \( \alpha > 0 \).

A government taxes labor in each sector and can condition the second household’s taxes on the sectoral choice of the first.

Figure 1 depicts the timing of household sectoral choice and its implications for output.

Remark. Economists often study Pigouvian taxation in contexts where households’ decisions are sensitive to the incentives they face on the margin. It may therefore surprise the reader that this model’s equilibria are corner solutions to the household problem, implying that household behavior is locally insensitive to marginal incentives. Nonetheless, Pigouvian taxation plays an important role in determining which of these corners can be supported.
2.2 Pigou’s conundrum

We begin by considering what can happen when the government sets taxes in the standard Pigouvian fashion. An Pigouvian equilibrium is a path for the wages faced by firms, the wages faced by households, and households’ labor supply and consumption decisions such that:

- The wages faced by firms are their marginal products of labor, i.e. $A$ in the traditional sector and $(L^I)^\alpha$ in the industrial sector.
- The wages faced by households are those set by firms plus the value of the marginal externalities they generate on output, local to the equilibrium path, i.e. $A$ in the traditional sector and $p_L I q^\alpha$ in the industrial sector.
- Taking these paths of wages as given, households make the labor supply decisions that maximize the NPV of their earnings.

Suppose full industrialization is Pareto efficient, i.e. $2A < 2^{1+\alpha}$. Can Pigouvian taxation support this outcome? Can it support other, inefficient, outcomes?

Pigouvian taxation can indeed support industrialization. On an equilibrium path in which both households industrialize, the post-tax industrial wage is $(1+\alpha)^2$. The fact that industrialization is efficient implies that this wage exceeds that offered in the traditional sector, rationalizing households’ equilibrium behavior:

$$2A < 2^{1+\alpha} \implies A < (1 + \alpha)^2.$$  \hspace{1cm} (1)

However, Pigouvian taxation can also support the “development trap” outcome where both households choose the traditional sector. This is simply because on the equilibrium path where no one works in the industrial sector, (a) productivity in the industrial sector is zero and (b) there is no one to benefit from externalities that boost industrial productivity. So the post-tax industrial wage is zero and both households prefer the traditional sector. The left panel of Figure 2 illustrates households incentives under these post-tax wages. If either household deviates from its equilibrium choice of the traditional sector then, holding the other household’s behavior fixed in the standard general equilibrium fashion, it ends up with a post-tax wage of zero instead of $A$. The dynamic structure of the diagram is unimportant for this point but will become relevant in the next section.

In other words, Pigouvian taxation cannot rule out the coordination failure outcome in which each household—expecting that no one will join it, should it migrate to the industrial sector—stays in the traditional sector. This self-fulfilling prophecy arises despite the fact
that taxes are set so as to correct for externalities on the margin of the equilibrium path where no one works in industry. The core problem is that non-industrialization corresponds to a local maximum of the production function, one from which neither household can move away without coordinating with the other.\(^3\)

As Pigou [1924] himself wrote, “All of the relative maxima are, as it were, the tops of hills higher than the surrounding country, but only one of them is the highest hill-top of all.”

### 2.3 An alternative to Pigouvian policy

The main contribution of this paper is to provide an alternative, “super-Pigouvian” policy that retains the spirit of Pigouvian policy while avoiding coordination failure. Within this simple model, I now introduce this policy in two steps.

First, consider the sectoral choice of the household who acts second. Our assumption that industrialization is efficient implies that it is efficient for this household to industrialize if the first household does. For simplicity, assume that it is otherwise efficient for the second household to choose the traditional sector, i.e. \(A > 1\).

How can a planner set taxes to guarantee that the second household chooses the efficient sector? One simple way is to “sell the household the economy” by ensuring that the difference in post-tax wages across the two sectors is equal to the difference in aggregate output that results from her choice. This way, she is a full residual claimant to her action’s effects and chooses the sector that maximizes output—i.e. whichever the first household has chosen.

The post-tax wages shown for the second household in the right panel of Figure 2 illustrate this logic. On one hand, if the first household chooses the traditional sector, then aggregate output is either \(2A\) if the second household also chooses the industrial sector or \(A + 1\) if the second household chooses the industrial sector. So if the second household’s post-tax wage in the traditional sector is \(A\), then its post-tax wage in the industrial sector should be \(1\).\(^4\) On the other hand, if the first household chooses the industrial sector, then aggregate output is either \(1 + A\) if the second household chooses the traditional sector or \(2^{1+\alpha}\) if it chooses the industrial sector. So if the second household’s post-tax wage in the traditional sector is \(A\), then its post-tax wage in the industrial sector should be \(2^{1+\alpha} - 1\).

Second, consider the sectoral choice of the household who acts first. How can the planner set taxes to guarantee it chooses the efficient sector? One method is to follow the same approach as for the second household, setting taxes so as to ensure that the first household’s

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\(^3\)Part of the problem is also that—due to the neoclassical assumption that households are price takers—each is unable to “coordinate with itself” and move to the industrial sector motivated by how doing so will affect its own productivity. However, the self-coordination issue disappears in the many household limit.

\(^4\)The level of the post-tax wage does not affect behavior because households have inelastic total labor supply. What matters here is the difference in post-tax wages across sector.
difference in post-tax wages across sectors is equal to the difference in aggregate output that results from her choice of sector. This is not quite as simple as in the case of the second household, since the level of output that results from any sectoral choice of the first household also depends on how the second household responds. In order for to incentivize the first household to industrialize, taxes must reflect the knowledge that the second household will follow suit—as the taxes we specified for it imply.\(^5\)

To see how this works, consider the first household’s post-tax wages shown in the right panel of Figure 2. If the first household chooses the traditional sector, then—as the second will also choose the traditional sector in this case—total output is \(2A\). If the first household instead chooses the industrial sector then—as the second household will also choose the industrial sector in this case—total output is \(2^{1+\alpha}\). So if the second household’s post-tax wage is \(A\) in the traditional sector, its post-tax wage in the industrial sector should be \(2^{1+\alpha} - A\).

To summarize, we have described a policy in which each household is paid a post-tax wage that reflects its total contribution to output, taking as given past behavior, and taking into account effects on future behavior. It is important to note that this can result in total post-tax wages that exceed aggregate output, since both households are full residual claimants of their effects on output. However, this does not present a challenge to the policy provided that the social planner can set lump-sum taxes to balance its budget after observing household behavior and—in the standard general equilibrium fashion—households take these transfers as given.

Note that this alternative policy retains elements of Pigouvian policy, as both regulate prices rather than quantities, and both compensate households for what is—in one sense or another—the contribution of their behavior to output. And, as we will later see, the incentives offered by both policies are closely related to households’ willingness to pay for one another’s actions. Motivated by these similarities, I call the alternative policy “super-Pigouvian.”

Super-Pigouvian policy differs from standard Pigouvian policy in two ways. First, it compensates households for the entire change in output induced by their action—inclusive of indirect effects through changes in later actors’ behavior. By contrast, Pigouvian policy only compensates household for the effect of changing their own action and does so using prices that reflect output contributions on the margin of the equilibrium path. Second, whereas Pigouvian policy need not be specified at nodes more than one deviation from the

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\(^5\)If, for instance, the first household believed that the second household would choose the traditional sector no matter what, then a policy that compensated the first household for its total effect on output would pay \(A\) in the traditional sector and 1 in the industrial sector. So the first household would choose the traditional sector.
equilibrium path, off-path super-Pigouvian policy disciplines what is meant by the “entire change in output” used in the definition of on-path Pigouvian policy.

The remainder of the paper formalizes, generalizes, and characterizes the idea of super-Pigouvian policy introduced informally above. Most importantly, I show that its ability to implement efficient outcomes is a general feature, rather than a quirk of this particular setting. Aside from this and other welfare properties, I also consider the information required for a planner to use Pigouvian or super-Pigouvian policy, that each can be implemented using knowledge of only households’ willingnesses to pay for one another’s choice of action. Finally I turn to a quantitative application that gauges the potential welfare gains from super-Pigouvian policy in practice.

3 Model

I now introduce a general model in which to formalize the ideas introduced in Section 2.

I take as my starting point a standard model of competitive equilibrium with production externalities and taxation. However, this model is not rich enough to accommodate super-Pigouvian policy because it has no notion of how households behave off of the equilibrium path. To capture this idea while remaining as close as possible to competitive equilibrium, I introduce the intuitive notion of “sequential competitive equilibrium”. In a sequential competitive equilibrium, the path of the economy is well-defined starting from any history of
household factor supply (like in subgame perfect Nash equilibrium) but this path is a competitive equilibrium in which households take prices as given.\footnote{“Sequential competitive equilibrium” has also been used to refer to competitive equilibrium in dynamic models in which households choose their consumption in each period (rather than at time zero), given their savings, e.g. Miao [2006]. This literature does not define behavior at all histories. My notion of sequential competitive equilibrium is a minor adaptation of the concept “subgame perfect competitive equilibrium” introduced by Nicolini [1992].}

3.1 Environment

The economy contains finitely many households $i \in \mathcal{I}$, finitely many factors $n \in \mathcal{N}$, one consumption good, and a finite or infinite number of discrete time periods $t \in \mathcal{T} = 0, 1, \ldots$

**Preferences:** At each time $t$, each household $i$ supplies factors $\ell^i_t$ and consumes $c^i_t$ units of the final good. Each household $i$ has GHH preferences $u(c^i_t - v(\ell^i_t))$ over consumption and factor supply within each period $t$ and discounts the future at a rate $\beta \in (0, 1)$. I assume $u$ is increasing, concave, and differentiable, and $v$ is increasing and concave.

While households can adjust their consumption freely, they are constrained in their ability to adjust their factor supply. Each household $i$ has initial factor supply $\ell^i_0$. At the end of each period $t$, exactly one household $i_t$ may update her factor supply to a new level $\ell^i_{t+1}$ contained in a feasible set $\mathcal{A} \subset \mathbb{R}^N$. In period $t + 1$, $i_t$’s supplies $\ell^i_{t+1}$ and all other households $j$ continue to supply $\ell^j_{t+1} = \ell^j_t$.

**Technology:** At each time $t$, a representative firm produces output $Y_t$ using factor inputs $L_t$, according to production function $Y_t = F(L_t, L_t)$. The first argument of $F$ represents inputs chosen by the firm, whereas the second represents a production externality that the firm takes as given. I assume that $F$ is differentiable and that it is constant-returns-to-scale and weakly concave in its first argument.

**Prices, taxes, and transfers:** At each time $t$, firms face a vector of factor prices $w_t$ and a final goods price that I normalize to one. In general, I allow the factor prices faced by each household $i$ to differ from the market wage by a vector of marginal taxes $\tau^i_t(\ell^i_t)$ that in general may depend on not only her identity but also her level of factor supply. I denote the corresponding post-tax factor prices by $\omega^i_t(\ell^i_t)$. Between periods, households have access to an exogenous interest rate $R > 1$, as in a small open economy. Finally, households receive lump-sum transfers $T^i_t$. 
3.2 Equilibrium

A sequential competitive equilibrium is a set of competitive equilibria, each corresponding to the economy’s path following a different history of factor supply. As in subgame perfect Nash equilibrium, I require that the competitive equilibrium following a time-(t + 1) history coincides with the competitive equilibrium starting from the time-t sub-history provided that households take equilibrium actions at time t.

More formally, A history \( h^t = (h_{t-1}, ..., h_0) \) is a sequence of factor supply updates, with each \( h_s \) representing the labor supply decision that \( i_s \) commits at time \( s \) to begin at time \( s + 1 \). I denote by \( h^0 = () \) the empty history before any households have acted.

**Definition 1.** A sequential competitive equilibrium is an on-path consistent profile of function \( \{c^t, \ell^t, Y, L, w, \omega^t, \tau^t, T^t\}_{t \in T} \) that, when applied to any history \( h^t \), returns a competitive equilibrium \( \{c_s^t(h^t), \ell_s^t(h^t), Y_s(h^t), L_s(h^t), w_s(h^t), \omega_s^t(h^t), \tau_s^t(h^t), T_s^t(h^t)\}_{t \in T} \) starting from that history. Formally, conditions (2)–(7) hold.

I now describe the equilibrium conditions for a competitive equilibrium in this model, as well as one additional condition that guarantees the sequential competitive equilibrium is self-consistent along the equilibrium path following any history.

First, the problem of household \( i \) at any history \( h^t \) is

\[
\{c_i^t(h^t), \ell_i^t(h^t)\} \in \arg \max_{\text{feasible } \{c_s, \ell_s\}_{s \geq t}} \sum_{s \geq t} \beta^{-(s-t)} u \left( c_s - v(\ell_s) \right) \\
\text{s.t.} \quad d^t(h^t) + \sum_{s \geq t} R^{-(s-t)} \left( c_s - \omega_s^t(\ell_s; h^t) \cdot \ell_s - T_s^t(h^t) \right) \leq 0, \tag{2}
\]

where \( d^t(h^t) \) and \( D(h^t) \) are household \( i \)'s and aggregate debt accumulated along the history \( h^t \) and where by “feasible” I mean that (a) \( \ell_s \in \mathcal{A} \), (b) \( \ell_{s+1} \neq \ell_s \) only if \( i = i_s \), and (c) \( \ell_t = h^t \) for last time \( t' \) before \( t \) at which \( i = i_{t'} \), or \( \ell_t = \ell_0 \) if no such \( t' \) exists. Above and throughout, I omit indices when clear from context, e.g. above \( \{c_i^t, \ell_i^t\} \) represents \( \{c_s, \ell_s\}_{s \geq t} \).

Second, following any history \( h^t \), the representative firm’s problem at any time \( s \geq t \) is

\[
Y_s(h^t), L_s(h^t) \in \arg \max_{Y, L} \ Y - w_s(h^t) \cdot L \quad \text{s.t.} \quad Y = F(L, L_s(h^t)). \tag{3}
\]

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Fixing \( h^t = (h_{t-1}, ..., h_0) \), let— for each \( i \in I - (\ell_i^t)_{s \leq t} \) be the path of labor supply defined by \( \ell_i^0 = \ell_i^t \), \( \ell_{i+1}^t = \ell_i^t \) if \( i \neq i_s \), and \( \ell_{i+1}^t = h_s \) if \( i = i_s \). Private and public debt \( d^t(h^t) \) and \( D(h^t) \) are given by

\[
d^t(h^t) = - \sum_{s < t} R^s \left( c_s^t(h^t) - \omega_s^t(\ell_s^t; h^t) \cdot \ell_s^t - T_s^t(h^t) \right)
\]

\[
D(h^t) = - \sum_{s < t} R^s \left( F \left( \sum_{i \in I} \sum_{s \leq t} \ell_i^s \right) - \sum_{i \in I} c_s^t(h^t) \right), \quad \text{where} \quad h^s = (h_r)_{0 \leq r < s},
\]
Third, following any history $h^t$, the schedule of factor prices faced by each household $i$ at any time $s \geq t$ differ from the prices facing the firm by the schedule of marginal taxes:

$$\omega^i_s(\cdot; h^t) = w_s(h^t) - \tau^i_s(\cdot; h^t).$$

(4)

Fourth, following any history $h^t$, factor markets clear at each time $s \geq t$:

$$L_s(h^t) = \sum_{i \in \mathcal{I}} \ell^i_s(h^t).$$

(5)

Fifth, following any history $h^t$, goods markets clear in net present value, taking as given aggregate debt accumulated up to $h^t$:

$$\sum_{s \geq t} R^{-(s-t)} Y_s(h^t) = \sum_{s \geq t} R^{-(s-t)} \sum_{i \in \mathcal{I}} c^i_s(h^t) + D(h^t).$$

(6)

Finally, in order for them to constitute a sequential competitive equilibrium, I require that the functions pertaining to each equilibrium quantity are consistent along the equilibrium path of factor supply following any history. That is, they make the same prescriptions at any history $h^t$ as at the histories that follow from $h^t$ on the equilibrium path—i.e. according to the function for factor supply. Formally, for any history $h^t = (h_{t-1}, ..., h_0)$,

$$x_{t+1}(h^t) = x_{t+1}((\ell^{i-1}_t(h^t), h_{t-1}, ..., h_0)),$$

(7)

for $x = c^i, \ell^i, \omega^i, \tau^i, T_i$ for all $i \in \mathcal{I}$ and for $x = w, Y, L$.

Moving forward, I refer to “sequential competitive equilibrium” as simply “equilibrium.”

### 3.3 Discussion

Two elements of the model warrant further discussion.

First, I have required that at most one household may act at a time. This assumption plays an important role in my results by narrowing the scope of coordination failure to cases where agents have self-fulfilling expectations about how others will act in the future. When actions are sequential, there can be no multiplicity due to self-fulfilling expectations about how others will act in the present. This assumption is innocuous in settings—such as worker migration—where, even if some actors move simultaneously, each is small enough so that

---

8The fact that goods markets need clear only in net present value—rather than in every period—reflects the presence of the savings technology (the source of the exogenous interest rate $R$) in the background. Together with households’ budget constraints, goods market clearing implies the government satisfies a lifetime budget constraint, which I therefore omit.
they do not affect optimality of the other’s decision. In settings—such as the formation of large firms—where this may not be the case, the essential aspect of my timing assumption is that, in contemplating her willingness to pay for j’s action, i has a well-defined sense of how j’s action will influence the behavior of others. Only to the extent that some other household k is likely to act at precisely the same time as j, so that j cannot influence k, does the super-Pigouvian approach breaks down.

Second, I have assumed that households’ consumption and factor supply strategies may condition on the entire history of factor supply, but may not condition on the history of consumption. While not critical, this assumption simplifies my analysis by allowing the planner to leave consumption un-taxed. I view it as sensible in the context of industrial policy, where, for example, workers looking to form beliefs about future employment in a sector are more like to refer to trends in sectoral employment rather than trends in aggregate consumption.

4 The limits of Pigouvian policy

I begin by analyzing standard Pigouvian policy. This serves to illustrate the strengths of Pigouvian policy as well as to bring its limitations into focus. In the next section, I will introduce a new policy that overcomes these limitations.

4.1 Defining Pigouvian policy

I begin by defining Pigouvian policy.

Definition 2. An equilibrium is Pigouvian if for all times $t$, households $i$, and factor supplies $\ell$,

$$\tau^i_t(\ell; h^0) = -F_T \left( L_t(h^0), L_t(h^0) \right).$$

(8)

This definition is standard: Factor supply subsidies should be set equal to a household’s marginal contribution to output through externalities, so that a household’s post-tax wages equals its total, marginal contribution to output. However, three details are worth highlighting. First, I have embedded Pigouvian policy in an environment that—unlike the standard neo-classical framework—explicitly models off-path behavior. Pigouvian policy does not specify how taxes are set off of the equilibrium path. Second, note that Pigouvian taxes only account for the externalities that a household generates directly, through its own behavior, and not any it generates indirectly by influencing the behavior of others. Finally, note that although marginal taxes in principle may depend on household identity and factor supply behavior, Pigouvian taxes do not.
4.2 Welfare properties of Pigouvian policy

Despite its inability to prevent coordination failure, Pigouvian policy has several positive welfare properties. As these properties are relatively well understood, I state them as a single result. I will then discuss how they can guide our thinking in the design of a policy that shares these properties while also addressing coordination failure.

These properties all relate to a weak notion of Pareto efficiency that I now define.

**Definition 3.** An allocation \( \{c^i_t, \ell^i_t\}_{t \in T} \) is **first-order Pareto efficient** if for all \( \{\Delta c^i_t, \Delta \ell^i_t\} \) such that \( \{c^i_t + \epsilon \Delta c^i_t, \ell^i_t + \epsilon \Delta \ell^i_t\} \) is feasible for small enough \( \epsilon > 0 \),

\[
\frac{d}{d\epsilon} \bigg|_{\epsilon=0} \sum_{j \in I} e^j \left( \{c^j_t + \epsilon \Delta c^j_t, \ell^j_t + \epsilon \Delta \ell^j_t\}_{t \in T}, \{c^j_t, \ell^j_t\}_{t \in T} \right) \leq 0
\]

where \( e^j(\{c^j_t, \ell^j_t\}_{t \in T}, \{c^j_t, \ell^j_t\}_{t \in T}) \) is the additional expenditure \( i \) would require in the second profile in order to achieve the same utility as it does under the first.

**Proposition 1** (Pigouvian welfare properties).

1. Every Pigouvian equilibrium is first-order Pareto efficient.
2. Suppose the action set \( A \) is convex. Then any first-order Pareto efficient allocation can be implemented as a Pigouvian equilibrium.
3. Suppose an equilibrium implements a first-order Pareto efficient allocation. Suppose that for some \( t, \ell^i_t(h^0) \) is interior to \( A \) and \( \omega_i(\ell, h^0) \) is differentiable in \( \ell \) for all \( s \) between \( t + 1 \) and the next time \( t' \) at which \( i_t \) adjusts. Then the taxes \( i_t \) faces between \( t + 1 \) and \( t' \) are—to first order local to \( i_t \)'s equilibrium action—Pigouvian, up to the re-shuffling payments across time.

In the second result above, and throughout the paper, I use “implement” in the Ramsey sense—i.e. to mean that there exists an equilibrium whose equilibrium path is the allocation in question—rather than the unique implementation sense from mechanism design.

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9By feasible, I mean that (a) labor supply is within \( A \) and satisfies its timing constraints and (b) goods market clearing holds in net present value when output is given by \( F \) evaluated at the labor supply allocation.
10Because preferences are GHH, we have

\[
e^j(\{c^j_t, \ell^j_t\}_{t \in T}, \{c^j_t, \ell^j_t\}_{t \in T}) = \sum_{l \in T} R^{t-l} \left( (c^j_l - v(\ell^j_l)) - (c^j_l - v(\ell^j_l)) \right).
\]

11On the surface, this assumption is somewhat restrictive in combination with our earlier assumption that \( v \) is convex, since it implies that, for example, a household who is indifferent between working in two different industries or locations must also be indifferent to splitting their time between them. However, it can be dispensed with by taking a limit in which there are many copies of every agent, a convexification “trick” that dates back to Aumann [1965]. For this reason I do not dwell on it.
The first result provides a fairly weak welfare guarantee for Pigouvian equilibria: They cannot be locally improved to first-order. A notable corollary is that, if the economy is convex in an appropriate sense—namely $A$ convex and $F(L, L)$ convex in $L$—first-order Pareto efficiency implies global Pareto efficiency, a first welfare theorem.\footnote{A version of this “Pigouvian first welfare theorem for convex economies” appears in Starrett [1972].}

So, where does Pigouvian taxation go wrong in non-convex economies? The second result provides an answer: Pigouvian taxation can implement any first-order Pareto efficient allocation—i.e. every “local optimum.” In non-convex economies, there may be many such allocations, only one of which is globally efficient. In other words, the shortcoming of Pigouvian policy is not its inability to support efficient outcomes, but rather its ability to rule out some inefficient outcomes—as we saw in the example of Section 2 (where the economy was non-convex). Pigouvian taxation runs the risk that households will land on a Pareto-dominated local maximum, i.e. will fall into a \textit{coordination failure}.

Given that Pigouvian policy implements not only global but also local optima, there is room for an alternative policy to do better. The third result narrows the scope for such improvements by clarifying that, \textit{along the path of any policy that implements efficient allocations}, policy must (at least locally) be set according to the Pigouvian rule.

This leaves room for two places in which an alternative policy might differ from Pigou. First, it can differ off of the equilibrium path. However, this cannot cause households—each of whom expect that the economy will remain on the equilibrium path whatever actions they take—to take different actions in the inefficient equilibria we would like to change. The second possibility is more subtle. On one hand, if we succeed in constructing an alternative policy that satisfies guarantees efficiency, then—as every equilibrium under that policy is efficient—the third result in Proposition 1 implies those equilibria must be Pigouvian. So the new policy must be Pigouvian in every strategy profile that is an equilibrium under the policy. On the other hand, Proposition 1 says nothing how taxes are set in hypothetical strategy profiles that are not equilibria. While, by definition, such profiles never arise in equilibrium, the fact that they are not equilibria may hold precisely because of the way the taxes that our alternative policy rule sets when applied to them. Given that the problem with Pigouvian policy is over- rather than under-inclusiveness, this is a promising approach.

Indeed, this is precisely the avenue taken by the “super-Pigouvian” policy introduced in the next section. Super-Pigouvian policy manages to set taxes so as to (a) be Pigouvian in efficient allocations and (b) prevent all other allocations from being consistent with household optimization. This resolves the coordination failure problem of Pigouvian policy while at the same time policy remains Pigouvian in any strategy profile that actually arises as an equilibrium.
5 Super-Pigouvian policy

In the previous section, I showed that Pigouvian policy can implement any Pareto efficient allocation—and moreover that any policy implementing a Pareto efficient allocation must effectively be Pigouvian along the equilibrium path—and yet Pigouvian policy can also implement Pareto inefficient allocations.

In this section, I propose an alternative policy that, unlike Pigouvian policy, always guarantees an efficient outcome—i.e. satisfies a first welfare theorem, even in non-convex economies. For lack of a better name, I call this super-Pigouvian policy.

5.1 Defining super-Pigouvian policy

In order to highlight the similarities between Pigouvian and super-Pigouvian policy, I begin by rearranging the definition of Pigouvian policy into a slightly different form. To start, since the Pigouvian post-tax wage is equal to a household’s entire marginal contribution over time, adding these contributions starting from any time \( t > 0 \) until the next time \( t' \) at which \( i_{t-1} \) can adjust implies

\[
\sum_{s=t}^{t'} R^{-(s-t)} \omega_s \ell_s(t-1)(h^0, h^0) \cdot \ell_s(t-1)(h^0) = \sum_{s=t}^{t'} R^{-(s-t)} \left. \frac{d}{dL} \right|_{L=L_s(h^0)} F(L, L) \cdot \ell_s(t-1)(h^0). \tag{11}
\]
Next, consider a change $\Delta \ell$ in $i_{t-1}$'s factor supply between $t$ and $t'$. In the limit where any individual household is small compared to the curvature of the production function, the change that this $\Delta \ell$ has on $i$'s NPV income between $t$ and $t'$ is

$$\approx \sum_{s=t}^{t'} R^{-(s-t)} F \left( L_s(h^0) + \Delta \ell, L_s(h^0) + \Delta \ell \right) - \sum_{s=t}^{t'} R^{-(s-t)} F \left( L_s(h^0), L_s(h^0) \right).$$  \hspace{1cm} (12)$$

Finally, note that if we add the effect that this $\Delta \ell$ has on $i_{t-1}$'s factor supply disutility between $t$ and $t'$, then the left-hand side equals her full perceived change in consumption-equivalent utility from deviating by $\Delta \ell$. Let $\Delta \tilde{U}^{i_{t-1}}(\Delta \ell, h^t)$ denote this change starting from an arbitrary history $h^t$. Similarly, if we add the change in $i_{t-1}$'s factor supply disutility to the RHS, then it represents the full effect of the $\Delta \ell$ deviation on the NPV of aggregate output less factor supply disutility over the entire future following $t$. This is because $\Delta \ell$ (a) only affects output until period $t'$ and (b) does not affect the labor disutility of any households other than $i_{t-1}$. Let $W(\{\ell_j^t\}_{s=t}^{t'})$ denote the NPV of aggregate output less factor supply disutility on any path of factor supply $\{\ell_j^t\}_{s=t}^{t'}$. Then Pigouvian policy ensures that for all histories $h^t = (\ell_{i_{t-1}}^t(h^0),...,\ell_{i_t}^t(h^0))$ on the equilibrium path—we have

$$\Delta \tilde{U}^{i_{t-1}}(\Delta \ell, h^t) \approx W \left( \left\{ \ell_i^t(h^t) + 1_{s\neq t'}^s \Delta \ell \right\} \right) - W \left( \left\{ \ell_i^t(h^t) \right\} \right)$$ \hspace{1cm} (15)$$

In words, Pigouvian policy ensures that, in adjusting their factor supply at a point along the equilibrium path, a household $i$'s perceived private incentive equals her effect on the NPV of aggregate output less labor disutility if all other households (including $i$ in the future) behave as if $i$ had played her equilibrium action.

The definition of super-Pigouvian policy below differs in two key ways. First, in measuring the social benefit of any deviation in factor supply, it accounts for how households who

\[ ^{13} \text{Formally, for any history } h_{s>0} = (h_{t-1},...,h_0), \text{ and any } \Delta \ell \text{ such that } h_{t-1} + \Delta \ell \in A, \text{ define} \]

$$\Delta \tilde{U}^{i_{t-1}}(\Delta \ell, h^t) \equiv \sum_{s=t}^{t'} R^{-(s-t)} \left[ \omega_s^{i_{t-1}}(\ell_{i_{t-1}}^s(h^t) + \Delta \ell, h^t) \cdot (\ell_{i_{t-1}}^s(h^t) + \Delta \ell) - v(\ell_{i_{t-1}}^s(h^t) + \Delta \ell) \right]$$ \hspace{1cm} (13)$$

$$- \sum_{s=t}^{t'} R^{-(s-t)} \left[ \omega_s^{i_{t-1}}(\ell_{i_{t-1}}^s(h^t), h^t) \cdot \ell_{i_{t-1}}^s(h^t) - v(\ell_{i_{t-1}}^s(h^t)) \right]$$

where $t'$ is the next time after $t - 1$ at which $i_{t-1}$ can adjust her factor supply (or $|T|$ if no such $t'$ exists).

\[ ^{14} \text{Formally,} \]

$$W(\{\ell_j^t\}_{s\geq t}) \equiv \sum_{s\geq t} R^{-(s-t)} \left[ F \left( \sum_{j \in I} \ell_j^s, \sum_{j \in I} \ell_j^s \right) - \sum_{j \in I} v(\ell_j^s) \right]$$ \hspace{1cm} (14)$$
act after any deviation \textit{change their behavior} in response, according to their factor supply strategies (which recall are defined off-path). Second, it considers the benefits of deviations away from all possible histories, not only those on the equilibrium path.

\textbf{Definition 4.} An equilibrium is super-Pigouvian if, for all histories $h^{t>0} = (h_{t-1}, ..., h_0)$ and for all $\Delta \ell$ such that $h_{t-1} + \Delta \ell \in A$,\footnote{Although this definition only \textit{implicitly} defines marginal taxes, one may rearrange it in order to obtain an explicit definition for marginal taxes using the definition of $\Delta \hat{U}^{t-1}$. Intuitively, $\Delta \hat{U}^{t-1}$ contains pre-tax wage payments, tax payments, and labor disutility; one may simply bring market wages and labor disutility to the other side of the equation. Note that taxes are still only defined up to the re-shuffling of payments over time between $t$ and the next period when $i_{t-1}$ may adjust her behavior.} \begin{equation}
\Delta \hat{U}^{t-1}(\Delta \ell, h^t) = W \left( \left\{ \ell^i_s \left( (h_{t-1} + \Delta \ell, h_{t-2}, ..., h_0) \right) \right\} \right) - W \left( \left\{ \ell^i_s (h^t) \right\} \right) \tag{16}
\end{equation}

The first essential difference between Pigouvian and super-Pigouvian policy is that the latter incentivizes households to change welfare not only directly, through the effects of their own actions, but also indirectly, through their actions’ influence on other households behavior. As in the motivating example of Section 2, this incentivizes households to behave as if they understood that their ability to change the behavior of others with whom they seek to coordinate. This helps prevent self-fulfilling expectations and coordination failure.

The second essential difference is that super-Pigouvian policy applies both on and off the equilibrium path, whereas Pigouvian policy only applies off of the equilibrium path. The role of this distinction is rather subtle, as—since households believe that (however they behave) prices and transfers will follow the equilibrium path—taxes set off of the equilibrium path do not affect behavior on the equilibrium path. To the extent that we are only concerned with efficiency along the equilibrium path, therefore, it would appear that the off-path definition of super-Pigouvian policy is irrelevant. However, the \textit{definition of super-Pigouvian policy on the equilibrium path depends on outcomes that occur off of the equilibrium path}, through off-path welfare. As a result, the precise way in which off-path welfare is constructed can affect the design of super-Pigouvian policy on path and therefore what allocation it implements. And—I will later show—the assumption that super-Pigouvian policy holds off of the equilibrium path is precisely the right way to discipline off-path welfare in order to guarantee that the policy succeeds on path.

The third essential difference between Pigouvian and super-Pigouvian policy is that, whereas (in the case of production externalities) the former only depends on production externalities, the latter depends on all impacts on production, as well as effects on factor supply disutility. The reason these terms do not appear in the Pigouvian case is that a household already internalizes its own direct effect on production through its wage as well
as its own direct effect on its factor supply disutility. This is no longer the case once one accounts for a household’s impacts on other households. 16

5.2 Avoiding coordination failure with super-Pigouvian policy

In Section 4, I showed that Pigouvian policy can implement any efficient allocation, but also runs the risk of implementing inefficient allocations if there is coordination failure. Above, I defined a new policy—super-Pigouvian policy—that differs from Pigouvian policy in that (a) it compensates actors for both their direct effects on welfare and their indirect effects, through impacts on other households’ behavior and (b) it requires that private and social incentives align both on and off the equilibrium path. Are these two modifications enough to rule out coordination failure while still supporting efficient equilibria?

Our next two results show that this is the case. First, super-Pigouvian policy satisfies a first welfare theorem.

**Theorem 1** (Super-Pigouvian first welfare theorem). If an equilibrium is super-Pigouvian, then it is Pareto efficient starting from any history.17

Both of super-Pigouvian policy’s distinguishing features—the consideration of indirect effects on welfare and the requirement that policy be set off path—play key roles in this result. The first feature ensures that policy effectively “sells the future of the economy” to the household $i$ who acts at $t$, by compensating her for all other households’ willingness to pay for the future her action creates. So $i$ acts in the public interest, taking as given her equilibrium belief about how the next actor will behave along the path she starts them on. The second feature ensures that this next actor, too, acts in the public interest, as do all who follow her. Since, at every history, these households act as the planner would, the principle of optimality from dynamic programming ensures the equilibrium path is efficient.

Apart from the design of super-Pigouvian policy, the other key building block of Theorem 1 is the assumption that only one household acts at each time. This rules out the possibility of coordination failure between two households acting simultaneously. With one household acting at a time and perfect alignment of household incentives with social welfare, it is as if a social planner simply hires one household to “run the economy” in each period.

16Pigouvian and super-Pigouvian policy also differ in a fourth, less essential, way that corresponds to the difference between the “$\approx$” in (13) and the “$\approx$” in (16). Namely, Pigouvian taxation compensates households for their marginal impact on production at existing quantities rather than their total impact. However, this difference vanishes in the large household limit, where any individual household’s action is small relative to the curvature of the production function.

17This result relies on an additional technical assumption: The function $F(\sum_{i\in I} \ell^i, \sum_{i\in I} \ell^i) - \sum_{i\in I} v(\ell^i)$ is bounded over across all $\{\ell^i\}_{i\in I} \in A^I$. 

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A potential concern with Theorem 1 is that it could hold vacuously by simply ensuring that no super-Pigouvian equilibria exist. My next result ensures that this is not the case.

**Theorem 2** (Super-Pigouvian second welfare theorem). If a strategy profile \( \{ c^i, \ell^i \} \) satisfying the on-path consistency condition (7) is Pareto efficient starting from any history, then it can be implemented as a super-Pigouvian equilibrium.

### 5.3 Informational requirements for super-Pigouvian policy

So far, I have shown that super-Pigouvian policy satisfies strong welfare properties, ruling out coordination failure and ensuring Pareto efficiency. However, skeptics will rightly point out that it is not the only such policy. For example, a planner could implement the efficient allocation by simply regulating quantities so as to make inefficient actions illegal.

When facing similar criticisms, advocates of standard Pigouvian policy can point out an additional advantage of Pigouvian policy: It can be implemented with limited information on the part of a social planner. In particular, a planner can evaluate whether an equilibrium is Pigouvian using only knowledge about the extent of externalities, by checking whether factor supply subsidies indeed align with factors’ marginal output contributions through the externality channel, and updating them if not. This does not require her to know the efficient allocation.

It is less obvious whether super-Pigouvian policy should satisfy a similar “test,” since its definition not only depends on more primitives but also on households’ endogenous behavior. My next result shows a social planner may nevertheless still verify whether taxes are super-Pigouvian using a simple test that only depends on households’ willingnesses to pay for one another’s factor supply adjustments.

**Proposition 2.** An equilibrium is super-Pigouvian if and only if for all histories \( h^{t>0} = (h_{t-1}, ..., h_0) \) and all alternative histories \( h^{t'} = (h_{t-1} + \Delta \ell, ..., h_0) \) with \( h_{t-1} + \Delta \ell \in \mathcal{A} \),

\[
\sum_{j \neq i_{t-1}} e^j \left( \left\{ c^j_s(h^{t'}), \ell^j_s(h^{t'}) \right\}_{s \geq t}, \left\{ c^i_s(h^t), \ell^i_s(h^t) \right\}_{s \geq t} \right) + \delta_{SP}(\Delta \ell, h^t) = 0, \tag{17}
\]

where \( \delta_{SP}(\Delta \ell, h^t) \) is a “correction” term that goes to zero in any limit in which households’ impacts on prices vanish.

In words, taxes are super-Pigouvian if and only if the aggregate willingness to pay of all households other than \( i_{t-1} \) for a change in her behavior is zero. This is the case when factor supply subsidies to \( i_{t-1} \)—which these households finance by accepting smaller lump-
sum transfers—exactly offset the other benefits of \( i_{t-1} \)'s actions to other households, making them indifferent.

This condition is akin to the willingness-to-pay condition often used as the definition of Pigouvian taxation for externalities that manifest in households’ utility functions (rather than production): Taxes on an action are Pigouvian if and only if they equal aggregate willingness to pay for that actions’ effect on utility through the externalities it generates, holding all other behavior constant. Proposition 2 provides a similar condition, but allows for willingnesses to pay to account for indirect effects—through impacts on other households’ behavior—and for all channels, rather than just the externality channel.

While super-Pigouvian policy can be implemented with relatively limited information on the part of a social planner, doing so requires that households have somewhat more information than neoclassical models assume. Typically, households are assumed to have rational expectations about the evolution of prices and transfers along the equilibrium path; this is what allows them to make optimal forward-looking decisions. However, for a planner to elicit from households their willingnesses to pay for one another’s behavior, households must also have rational expectations about how prices and transfers will evolve off of the equilibrium path. This is a more demanding requirement, more similar to what is typically assumed in dynamic games.

Note, however, that this does not imply that any individual household is so knowledgable that, on its own, it knows the efficient path of the economy. Households only need know only the equilibrium path of prices and transfers starting from any history—not the path of aggregate welfare. In this sense, the economy still aggregates information, achieving an allocation that no household could on its own.
5.4 Extensions

With an eye toward the quantitative application presented in the next section, I now explain how my characterization of super-Pigouvian policy extends to environments with various forms of heterogeneity, uncertainty, and imperfect competition. The same extensions also apply to my characterization of Pigouvian policy.

**Heterogeneity and time variation:** The model of Section 3 assumes that all households have the same intra- and inter-temporal preferences over consumption and labor supply. Additionally, it assumes that the production function and the interest rate are constant over time. All of my results continue to hold in the more general model where:

- Each household $i$ has a discount factor $\beta_i \in (0, 1)$, time-varying factor supply disutility functions $\{u_i^t\}_{t \in T}$, time-varying consumption utility functions $\{u_i^t\}_{t \in T}$, and time-varying action sets $\{A_i^t\}_{t \in T}$. Preferences are still GHH and satisfy the same technical properties, and action sets are consistent with the timing of behavior.\(^\text{18}\)
- The production function $F_t$ varies over time. The production function still satisfies the same technical properties within each period.
- The interest rate $R_t > 0$ varies over time. The quantity $\sum_{t=0}^{\infty} (R_t)^{-t}$ is still bounded.

**Uncertainty:** The model of Section 3 is completely deterministic. All of my results continue to hold in the more general model where:

- Household preferences, the production function, and the function $\iota(\cdot)$ that determines the timing of household actions are all stochastic processes, measurable with respect to a common filtration.
- All households have accurate beliefs about these processes, and all uncertainty is public—there is no imperfect/asymmetric information.
- Households have access to perfect insurance at exogenous and risk-neutral prices.

In this model, all equilibrium variables—including taxes—are interpreted as filtration-measurable functions of the underlying state of the world.

**Imperfect competition:** It is well known that models of imperfect competition can be represented in reduced form as models of perfect competition with external economies of scale—i.e. externalities—when firms are monopolistically competitive within each sector and face CES demand [Kucheryavyy et al., 2016]. Below, I provide a new “recipe” for reinterpretting imperfect competition as externalities under far more general assumptions.

\(^\text{18}\)That is, if $i$ does not act between $t$ and $t'$, then $A_i^t = A_i^{t'}$. 

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Specifically, I require that firm entry is competitive, but allow for arbitrary forms of competition among existing firms.

- Households’ factor supply decisions are interpreted generally, so as to include decisions such as operating a firm of a particular type. Similarly, households’ factor incomes are interpreted generally, so as to include income sources such as profits. Households take as given the post-tax income schedule that corresponds to these decisions.\(^{19}\)
- To the extent that any household has multiple sources of income—such as labor income and dividend income—these can be taxed jointly and with quantity-dependence.
- There exists a (differentiable) per-unit-supplied income function \(\hat{w}(\cdot)\) such that at any factor supply profile \(\{\ell_t^i\}_{i \in I}\), each household \(i\) has pre-tax income \(\hat{w}(\sum_{j \in I} \ell_t^j) \cdot \ell_t^i\).
- At every factor supply profile, aggregate output equals aggregate pre-tax income.

Under these assumptions, the production side of the economy can simply be reinterpreted as deriving from a perfectly competitive model with externalities. In particular, the production function \(F\) of this perfectly competitive economy may be defined as

\[
F(L, \bar{L}) \equiv \hat{w}(\bar{L}) \cdot L. \tag{18}
\]

6 Quantitative application

So far, I have compared the design and welfare properties of Pigouvian and super-Pigouvian policies in a general, theoretical setting. This leaves open an important practical question: Could super-Pigouvian industrial policies produce quantitatively meaningful welfare gains?

I now address this question by assessing the impacts of super-Pigouvian policy in a calibrated dynamic model of structural transformation in a small open economy. This model retains the basic spirit of the motivating example in Section 2 but captures several key institutional features that make it possible to calibrate empirically. In the model, entrepreneurs decide whether to operate a new, industrial firm or to instead work in the traditional sector. The key distinction between these sectors is that the industrial sector uses the final good that it creates as an input. This feedback makes entrepreneurs’ entry complementary and can generate multiple equilibria. The model fits into the extensions described in Section 5.4, so my earlier theoretical results apply.

\(^{19}\)For example, this formulation does not allow a household to open fewer firms out of concern that her marginal firm will decrease the profits of her inframarginal firms. However, it does allow these firms to themselves make imperfectly competitive choices—such as marking up prices or marking down wages.
I calibrate this model using South Korea’s targetted industrial policy to the heavy and chemical industries—which I interpret as the “industrial sector”—in the 1970s. In particular, I interpret the rapid expansion of these industries as a transition between steady states and use this assumption to back out the parameters of my model from statistics reported in Lane [2022]. Finally, I simulate the equilibria of the calibrated model in order to assess the gains from supre-Pigouvian policy.

6.1 Model

Motivated by my application to South Korea in the 1970s, I develop a model of a structural transformation featuring an industrial sector with strong complementarities between firms due to roundabout production. I interpret the industrial sector as a reduced form representation of the heavy and chemical industries (HCI), which are both input intensive and used heavily as inputs, and which were directly targetted during by “HCI drive” policies between 1973 and 1979. The static model I build is most similar to Rodrik [1996], but—as stagegered actions over multiple time periods are essential to super-Pigouvian policy—I embed it in a dynamic setting where firms’ entry decisions are inertial. Similarly to Frankel and Pauzner [2000], I assume that productivity in the industrial sector follows a stochastic process, with the goal of “smoothing out” the responsibility of individual households for dramatic transitions between steady states. However, I depart from Frankel and Pauzner [2000] by assuming that productivity shocks are mean-reverting, so as to ensure that the model can generate equilibrium multiplicity and, therefore, coordination failure.20

6.1.1 Environment

Workers and entrepreneurs interact in a traditional and an industrial sector over a discrete, infinite \( T \) spaced an amount \( \Delta \). At any time \( t \), the economy consists of one final good and a measure \( N_t \) of intermediate varieties.

Households: The population contains three types of households: legacy industrialists, workers, and entrepreneurs. While agents of each type are “households” in the sense of the general model in Section 3, the entrepreneurs are the main economic actors of interest and those whose behavior super-Pigouvian policy can influence in an interesting way.

First, a representative, immobile legacy industrialist owns and operates \( N \) industrial firms that produce differentiated varieties.

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20Frankel et al. [2005] show that mean-reversion is consistent with equilibrium uniqueness if it only lasts for a certain number of periods. I assume the productivity process has a constant level of mean-reversion over an infinite horizon.
Second, a representative worker inelastically supplies $M\Delta$ total units of labor in each period. Each infinitesimal unit $m$ of this labor has productivity 1 in the industrial sector and productivity $a(m)$ in the traditional sector, distributed according to the CDF

$$P[a(m) \leq a] = \max\left[\left(\frac{a}{\bar{a}}\right)^{\varepsilon}, 1\right]$$

for some $\varepsilon, \bar{a} > 0$. I impose parametric assumptions that guarantee $\nu_t \leq \bar{a} \chi_t$. This implies supply curves for industrial labor $L^I_t$ and efficiency units of traditional labor $L^T_t$:

$$L^I_t = M \left(\frac{w^I_t}{\bar{a} w^T_t}\right)^{\varepsilon}, \quad L^T_t = \frac{\varepsilon}{\varepsilon + 1} M \left(\bar{a} - \frac{(w^I_t/w^T_t)^{1+\varepsilon}}{\bar{a}^{\varepsilon}}\right)$$

where $w^I_t$ and $w^T_t$ are the industrial and traditional sector wages per effective unit of labor.

Third, a finite set of entrepreneurs $i \in \mathcal{I}$ is endowed with $I/|\mathcal{I}|$ units of effort per period of which she can devote some or all to operating variety producers. Each entrepreneur operates a unit measure of variety producers per unit effort supplied. Entrepreneurs are imperfectly mobile. Concretely, each $i \in \mathcal{I}$ may adjust her entry decision at the beginning of any period $t$ if and only if $\iota(t) = i$, where $\iota(t)$ is a scalar random variable. $\iota(t)$ is distributed independently across time and symmetrically across mobile households, taking each value $i$ with probability $\delta \Delta < 1/|\mathcal{I}|$.\(^{21}\)

Intertemporally, households of all types save at an exogenous interest rate and buy insurance in perfect and risk-neutral markets in order to maximize expected lifetime utility

$$E \left[ \sum_{t \in \mathcal{T}} \beta^{-t} u(c_t) \Delta \right],$$

where $u$ is increasing, strictly concave, and differentiable. Households’ consumption strategy—as well as their migration decisions—may depend not only on the history of migration decisions, but also on the history of the random process for mobility, $\iota(t)$, and the random process for productivity, $z_t$, introduced below.

**Production:** In the traditional sector, a representative, competitive firm produces the final good one-for-one with effective units of labor and pays a competitive efficiency wage $w^T_t$.

At any time $t$, the industrial sector contains two types of firms: $N_t$ monopolistically competitive producers of differentiated varieties and a representative, competitive final goods producer that aggregates these varieties.

At each time $t$, each intermediate variety $n$ producer pays a (flow) fixed cost $f$ in units of

\(^{21}\) $\iota(t)$ can also take the value 0, in which case no household may migrate.
the final good and produces output \( q_t(n) \) according to a Cobb-Douglas production function

\[
q_t(n) = l_t(n)^\alpha x_t(n)^{1-\alpha},
\]

where \( \alpha \in (0, 1) \) is the labor share, \( l_t(n) \) is labor, and \( x_t(n) \) is the final good (as an intermediate input). Each intermediate firm buys inputs competitively at wages \( w_t^I \) and final goods price 1, but sets its selling price \( p_t(n) \) to maximize profits given the final goods producer’s demand function for inputs. The firm therefore earns profits

\[
\pi_t(n) = p_t(n)q_t(n) - x_t(n) - w_t^I l_t(n) - f.
\]

(23)

I assume that variety producers must operate if they exist, even if they earn negative profits.

At each time \( t \), a representative, competitive final goods producer produces gross output \( Q_t \) according to a CES production function

\[
Q_t = z_t \left[ \int_0^{X_t(n)} X_t(n)^{\frac{\eta-1}{\eta}} \, dn \right]^{\frac{\eta}{\eta-1}},
\]

where \( \eta > 1 \) is the elasticity of substitution across varieties, \( X_t(n) \) is the quantity of each variety \( n \) used as an input, and \( z_t \) is a productivity shock. I assume \( \log z_t \) follows an AR(1) process

\[
\log z_t = \log z_{t-1} + \theta \Delta (\mu - \log z_{t-1}) + \sigma \sqrt{\Delta} \epsilon_t
\]

(25)

where \( \theta, \sigma > 0 \) and \( \epsilon_t \sim N(0, 1) \) i.i.d. I normalize the price of the final good to one.

**Policy:** A government taxes and subsidizes the total labor income of each household, but not how each household’s labor or firm’s intermediate inputs are allocated in the market. Both taxes and transfers may condition not only on the history of migration decisions, but also on the history of the random processes \( \iota(t) \) and \( z_t \) and household identity. The government saves and trades in insurance so as to balance its budget intertemporally.

**Market clearing:** In each period, labor markets clear in both the traditional and industrial sectors, input variety markets clear, and final goods markets clear.

### 6.2 Calibration

#### 6.2.1 Calibration strategy

I calibrate this model using estimates from Lane [2022] of the effects of South Korea’s heavy and chemical industrial (HCI) drive from 1973 to 1979. This policy provided large
and targetted subsidies to these industries in the form of subsidized lending, as well as exemptions from trade restrictions and tariffs on imports. The main idea behind this strategy is to interpret the sharp and persistent increase in HCI output that followed as a transition between the model's steady states under Pigouvian policy.

Three main assumptions underlie this approach. The first is that policy was Pigouvian before and after the HCI drive. Starting under the government of Park Chung-hee in the early 1960s, South Korea engaged in an industrial policy of broad export promotion. In constrast to the HCI drive, this policy was not targeted toward any particular sector. Following Park's assassination in 1979, the HCI drive was ended and South Korea returned to a broad (if somewhat accelerated) strategy of export promotion [Lane, 2022]. To the extent that one thinks of all exporting sectors as experiencing similar external economies, these broad policies can be intepretted as Pigouvian to a first approximation.

My second main assumption is that the change in economic activity in the heavy and chemical industries during the course of the HCI drive can be intepreted as a shift to a new equilibrium at the same fundamentals—rather than a change in fundamentals. The main competing narrative is that temporary subsidies to these industries led to entry in the short term, which in turn increased productivity in the long run. However, two observations support the multiple equilibrium interpretation: First, Lane [2022] documents that growth in HCI employment, the number of HCI firms, and HCI output per worker—relative to their non-HCI counterparts—followed very similar time paths, growing markedly during the HCI drive and then stabilizing immediately afterwards. If growth in output per worker reflected learning-by-doing, one would expect it to lag growth in the “learners”, i.e. workers or firms. Second, the growth in output per worker that Lane [2022] documents need not be explained by intra-firm productivity growth, as in a learning-by-doing model. Indeed the model presented above also predicts growth in output per worker due to the reduced input prices that result from firm entry. In fact, my calibration predicts ~40% growth in output per worker, close to what Lane [2022] documents.22

The third key assumption behind my calibration is that—granted the HCI drive led to a shift toward a new steady state—it is appropriate to interpret post-period observations as defining that steady state. This entails assuming both (a) that any stochastic fluctuations of the steady state around a particular point can be ignored and (b) post-period observations occur at the new steady state rather than at a point along the transition path toward that steady state. To address (a), I assume that productivity fluctuations are small enough that

22I show in Appendix B.4 that log growth in output per worker is equal to $\frac{1}{\alpha(\eta-1)}$ times log growth in the number of firms. I calibrate $\alpha = 0.31$ and $\eta = 5$, and Lane [2022] documents ~0.5 log growth in the number of plants (which I interpret as firms in the model).
they do not meaningfully affect output in equilibrium, setting $\sigma = 0.03$. This is less of an empirical stance and more of a choice to focus on the low-noise limit.\footnote{I verify that, among similarly low values of $\sigma$, my results are not sensitive to its precise value.} Moreover, I take the mean-reversion of productivity, to be relatively high, setting $\theta = 3$. This, too, is an interpretational choice rather than an empirical one, motivated by a desire to study dynamic multiplicity and the finding of Frankel and Pauzner [2000] that there is a unique dynamic stochastic equilibrium when productivity follows a random walk.\footnote{While on one hand, my calibration of $\theta$ and $\sigma$ is somewhat arbitrary, it has a natural and plausible interpretation: there are small and temporary shocks to demand.} To address (b), I calibrate $\delta = \frac{1}{7}$ so that most entrepreneurs have a chance to adjust during the HCI drive between 1972 and 1979. Here, I draw on the fact that the majority of disproportionate growth in HCI (rather than other sectors) happened by the end of the HCI drive in 1979.

### 6.2.2 Calibration details

I now explain how I calibrate each parameter of the model. In cases where the identification of a parameter from moments in the data requires significant rearrangement of the model’s equilibrium conditions, I simply describe which moments are used and relegate the algebra to Appendix B.4. Except where I state otherwise, I take moments from estimates in Lane [2022], using his 4-digit panel of HCI firms.

I set the number of legacy industrialists $N$ equal to the average number of HCI firms and HCI workers between 1967 and 1972. I set the mass of entrepreneurs $I$ equal to the number of entrants into HCI between 1972 and 1979. I set the elasticity of substitution across input varieties $\eta \approx 5$ in order to match the increase in HCI gross output per worker between 1972 and 1979, given $\alpha$. This estimate is squarely within the range generally estimated in the trade and industrial organization literatures [Broda and Weinstein, 2006, Hendel and Nevo, 2006]. I set the labor share of gross output $\alpha$ equal to its value in the BEA’s 2012 use table in the US for the industries I designate as HCI.\footnote{These are: Primary metals; Fabricated metal products; Machinery; Computer and electronic products; Electrical equipment, appliances, and components; Motor vehicles, bodies and trailers, and parts; Other transportation equipment; Petroleum and coal products; and Chemical products.} I set the elasticity of labor supply into the industrial sector $\varepsilon$ to match the ratio of growth in the number of plants and growth in employment during the HCI drive. I back out the constant in the labor supply equation, $M/\bar{a}^\varepsilon$, from the industrial labor supply curve, $L_I^t = M\bar{a}^{-\varepsilon}(w^t_I)^\varepsilon$.\footnote{The separation of $M/\bar{a}^\varepsilon$ into $M$ and $\bar{a}$ is undetermined but unnecessary for my purposes, except for that I maintain the assumption that $\bar{a}$ is sufficiently large; see Appendix B.2.} I set the mean of the productivity process $\mu$ to match total HCI output per worker, given the number of firms, before the HCI drive. To set fixed costs $f$, I use the observation that entry must be un-profitable before the HCI drive and profitable afterwards. This does not completely determine $f$, so I make

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*I maintain the assumption that $\bar{a}$ is sufficiently large; see Appendix B.2.*
Table 2: Calibration summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Strategy</th>
<th>Data source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>5,354</td>
<td># HCI establishments before HCI drive</td>
<td>Lane [2022]</td>
</tr>
<tr>
<td>$I$</td>
<td>$N(\exp(0.5) - 1)$</td>
<td>$N \times$ (% increase in HCI plants during HCI drive)</td>
<td>Lane [2022]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>5</td>
<td>Match HCI output / worker during HCI drive</td>
<td>Lane [2022]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.31</td>
<td>Share of HCI inputs in HCI gross output</td>
<td>BEA (2012)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>1.23</td>
<td>Match change in HCI emp. / plant during HCI drive</td>
<td>Lane [2022]</td>
</tr>
<tr>
<td>$M\bar{a}$</td>
<td>27,409</td>
<td>Match HCI employment before HCI drive</td>
<td>Lane [2022]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-0.78</td>
<td>Match HCI output / worker given $N$, before HCI drive</td>
<td>Lane [2022]</td>
</tr>
<tr>
<td>$R$</td>
<td>1.1</td>
<td>Match interest rates during HCI drive</td>
<td>Kim [1991]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1/7</td>
<td># adjustments during HCI drive $= I$</td>
<td>-</td>
</tr>
<tr>
<td>$f$</td>
<td>605</td>
<td>Entran losses before HCI drive $= \frac{1}{2}$ (entrant profits after)</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.03</td>
<td>Illustrate multiplicity</td>
<td>-</td>
</tr>
<tr>
<td>$\theta$</td>
<td>3</td>
<td>Illustrate multiplicity</td>
<td>-</td>
</tr>
</tbody>
</table>

an assumption biased somewhat in favor of industrialization’s efficiency and set $f$ so that entrant losses in the pre-period are equal to half of entrant profits in the post-period. I set the interest rate $R = 1.1$, an approximation of bank interest rates in South Korea over this period [Kim, 1991].

Table 2 summarizes the calibration.

Under this calibration, I simulate the model in continuous time, many agent limit as $\Delta \to 0$ and $|Z| \to \infty$. This allows me to use efficient methods based on Moll [2017]. As there are many possible equilibria, I focus on the most-industrialized and least-industrialized equilibria—both under Pigouvian policy and laissez faire—as well as the social optimum—or equivalently the super-Pigouvian equilibrium. Here I leverage the fact that, since my calibration satisfies $\alpha(\eta - 1) < 1 + \varepsilon$, profits are increasing in the number of firms; a monotonicity argument in Frankel and Pauzner [2000] then implies that these extremal equilibria exist and take the form of entry cutoffs in the space of $z_t$ and $N_t$. 

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6.3 Results

Using the calibration and computational approach discussed above, I compute the least-industrialized and most-industrialized Pigouvian equilibria, as well as the efficient strategy—which recall is a super-Pigouvian equilibrium. I discuss these findings positively and normatively below.

6.3.1 Pigouvian multiplicity

I focus on the class of equilibria in which the entry decision depends on a cutoff threshold in the space of productivity and the current number of entrepreneurs. Frankel and Pauzner [2000] show that the equilibria with the most and least industrialization always take this simple, cutoffs form. Concretely, in such an equilibrium there exists a function \( z(N) \) such that entrepreneurs at \( t \) operate if and only if \( z_t > z(N_t) \). An equilibrium with low cutoffs reflects that entrepreneurs are to open a firm even when the exogenous component of productivity is low, because they expect many other entrepreneurs to join the market soon. Conversely, an equilibrium with high cutoffs reflects that entrepreneurs are pessimistic about whether other entrepreneurs will join the market and so are unwilling to open firms unless the exogenous component of productivity is very high.

Can these expectations about other entrepreneurs be so important so as to be self-fulfilling—creating a coordination failure? I find that this is indeed the case. Figure 4 demonstrates this by plotting the equilibrium cutoffs that define the most- and least-industrializing equilibria, i.e. those in which the productivity cutoffs for entrepreneurs to operate is the lowest and highest, respectively. These cutoffs at low values of entry, including at the point—where the dotted line representing mean log productivity intersects the \( x \) axis—that represents the initial condition of the HCI drive. Since the stochastic process for log productivity is fairly stable, with its stationary distribution having a standard deviation of \( \sigma/\sqrt{2\theta} = 0.012 \), it is easy to interpret the path of the economy under these two equilibria: Following the lowest-HCI Pigouvian equilibrium, the economy (with high likelihood) stays at its initial level of HCI firms with. Following the, highest-HCI Pigouvian equilibrium, all available entrepreneurs enter the heavy and chemical industries.

The figure also plots the cutoffs that describe the first-best equilibrium. As there is a unique first-best strategy profile, Theorem 1 implies that these are the equilibrium cutoffs in any super-Pigouvian equilibrium. Interestingly, the efficient cutoffs coincide with the most-industrializing Pigouvian equilibrium. In other words—although this need not be the case in general—it is under this calibration efficient for households to have the highest level of industrial optimism that Pigouvian policy can sustain. Appendix Figure A1 shows a version
Figure 4: Productivity cutoffs for entrepreneurial entry characterizing the highest- and lowest-industrializing equilibria of the calibrated model under Pigouvian policy.

of the figure that also contains (the unique) laissez-faire equilibrium

6.3.2 Welfare gains from super-Pigouvian policy

I next consider welfare in each of the three equilibria depicted in Figure 4. Since labor supply is fixed and the planner has access to lump-sum transfers, welfare is simply the net present value of aggregate output. In principle, one could compare output across the three equilibria starting from any \( (z_t, N_t) \) initial condition. However, the calibrated productivity process is sufficiently concentrated around its mean that it suffices to simply perform this exercise while setting \( \log z_t = \mu \).

Figure 5 shows welfare gains—compared to laissez faire—from Pigouvian policy in the lowest- and highest-industrialization equilibria. Recall that the latter coincides with the welfare gains from super-Pigouvian policy. I consider these gains as a share of the annuitized value of HCI value added before the HCI drive. A value of 1% is therefore the welfare equivalent to a value added increase of 1% in the heavy and chemical industries.

The figure shows that, depending on the initial share of non-legacy entrepreneurs who are active, the efficient allocation improves upon laissez faire by between between 1.8% and 9.9% of HCI value added. These gains are smaller when there are fewer firms initially, as in this case the (usually) efficient path—industrialization—is only slightly more efficient than the typical laissez-faire path—de-industrialization.

Most interesting is how welfare under the lowest-industrialization Pigouvian equilibrium
Figure 5: Welfare gains from the least-industrializing Pigouvian equilibrium and the efficient, most-industrializing Pigouvian equilibrium, at log productivity $\mu$ and various initial shares of non-legacy entrepreneurs entered.

compares to the laissez-faire and efficient welfare levels. Namely, whether this equilibrium delivers the laissez-faire welfare level or the efficient welfare level depends on the initial share of non-legacy entrepreneurs who are active. When there are sufficiently many, even the low-industrialization Pigouvian path leads to the efficient path, industrialization (with high likelihood); when there are too few, it leads to, it leads to the inefficient path, deindustrialization (with high likelihood).

The initial conditions of my calibration correspond to the left-most points in the figure. Here, the worst Pigouvian equilibrium is worse for welfare than the efficient equilibrium, implying there are gains from super-Pigouvian policy. However, these gains are modest, equivalent to about two percent value added in HCI industries.

6.3.3 Super-Pigouvian policy in inefficient Pigouvian equilibria

As discussed in Sections 4 and 5, super-Pigouvian policy is simply Pigouvian in any efficient equilibrium. However, one benefit of the calibrated model is that I can use to study the structure of super-Pigouvian policy in inefficient allocations—such as the lowest-HCI Pigouvian equilibrium discussed above. This both helps to clarify the mechanism behind super-Pigouvian policy and to shed light on whether a social planner can credibly commit to offer super-Pigouvian incentives.

Figure 6 compares Pigouvian and super-Pigouvian subsidies to HCI firms in the ineffi-
cient, lowest-HCI Pigouvian equilibrium discussed in the preceding sections. As in earlier sections, I study these subsidies at a range of values for the share of non-legacy entrepreneurs but assume log productivity is at its mean, \( \mu \). Intuitively, Pigouvian policy is increasing in the number of HCI firms—i.e. who it is that an additional firm can have an externality on—and increases sharply when around 20% of non-legacy entrepreneurs are active, since this is the threshold above which the future actors will choose to enter (see Figure 4).

Super-Pigouvian policy coincides with Pigouvian policy almost exactly except for in a narrow region around the 20% threshold. Intuitively, this is because this cutoff is inefficiently high, and super-Pigouvian policy can prevent it from being consistent with individual optimization if it offers households a large incentive to enter at a slightly lower threshold.

Although this is a broad feature of super-Pigouvian policy—that it offers strong incentives for coordination in a critical region—the strength of these incentives is particularly dramatic in this example, with the magnitude of the super-Pigouvian subsidy dwarfing the Pigouvian subsidy around the critical threshold (if not truncated, the graph would show a maximum around 18,000). This extremeness is undesirable from the perspective of super-Pigouvian policy’s practical implementability for two reasons: First, the government may not be able to commit to make sure large payments. Second, the government may not be able to acquire precise enough information to target the critical region, given it is very narrow. To the first point, it is important that, in fact, firms would be willing to enter with much smaller, discounted subsidies until their next adjustment exceed 1730 Won. In this sense, the super-Pigouvian planner has no need to provide such generous subsidies. It wishes to, because the welfare contributions of the pivotal household are so large, but household behavior would be the same with subsidies that are capped at a much lower level. To the second point, it is worth noting that a “smoothed out” policy that spreads the super-Pigouvian subsidy over a larger region would still prevent the low-industrialization Pigouvian allocation from being an equilibrium, even if it created a small amount of misallocation on the margin.

7 Conclusion

Coordination failures are one of the main motivations that policymakers cite for engaging in industrial policy. Yet, there is no clear policy solution: Pigouvian taxes that address market failures on the margin risk arriving at a local, rather, than global optimum—whereas its known alternatives require a centralized approach based on regulating quantities rather than prices.

This paper has proposed an alternative, “Super-Pigouvian” policy with the potential to
Figure 6: Subsidies to HCI firms under Pigouvian and super-Pigouvian policy, at log productivity $\mu$ and various initial shares of non-legacy entrepreneurs, in the lowest-HCI Pigouvian equilibrium. Y axis is expected discounted subsidies until next readjustment, in thousands of 2010 real Won.

resolve coordination without requiring centralized information. The idea behind this policy is to elicit households willingness to pay for each others’ actions—as in Pigouvian policy—taking into account those actions’ indirect effects on others’ later behavior. When households know that policy has this structure both on and off the equilibrium path, they are guaranteed to take the same actions the social planner would. This guarantees Pareto efficiency.

While this paper has studied the application of the super-Pigouvian approach to industrial policy, there is room to apply similar ideas to other settings. At a high level, this paper’s method for deriving welfare theorems—one that leans on arguments from dynamic programming rather than convex optimization—may be useful in applications where convexity is unlikely to hold. For instance, the literature on economic geography actively debates whether or not agglomeration forces lead to multiple equilibria, but has not yet proposed new policies that can ensure multiplicity is resolved efficiently [Bleakley and Lin, 2012, Allen and Donaldson, 2020]. This would be a natural setting for future work.
References


Appendix

A Proofs

A.1 Lemmas characterizing Pareto efficiency, household optimality

Lemma 1. Fix an equilibrium and a history $h^t$. The equilibrium profile is Pareto efficient starting from $h^t$ if and only if its path of factor supply $\{\ell_t^j\}_{j \in I}$ maximizes $W(\{\ell_t^j\}_{j \in I})$ among feasible allocations starting from $h^t$.

Proof. Formally, a profile $\{c_s^j, \ell_s^j\}_{s \geq t}$ is Pareto efficient starting from $h^t$ if and only it satisfies the following conditions:

- For all $j \in I$, we have (a) $\ell_t^j \in A$ for all $s \geq t$, (b) $\ell_t^j = h^j_t$ for the last time $t^j$ before $t$ at which $i^j_t = j$, or $\ell_t^j = \ell_{t_0}^j$ if no such time exists, and (c) for all $s \geq t$, $s < |T|$, we have $\ell_s^j = \ell_{s+1}^j$ unless $j = i_s$.
- $\sum_{s \geq t} R^{(s-t)} \left[ \sum_{j \in I} c_s^j - F \left( \sum_{j \in I} \ell_s^j, \sum_{j \in I} \ell_s^j \right) \right] + D(h^t) \geq 0$.
- Among all other profiles satisfying the first two bullets, none weakly improve welfare for all households and strictly improve welfare for at least one.

Since utility is non-satiated, these Pareto efficiency conditions are equivalent to

$$\begin{align*}
\{c_s^j, \ell_s^j\}_{s \geq t} &\in \arg\min_{\{c_s^j, \ell_s^j\}_{s \geq t} \text{ feasible}} \sum_{s \geq t} R^{(s-t)} \left[ \sum_{j \in I} c_s^j - F \left( \sum_{j \in I} \ell_s^j, \sum_{j \in I} \ell_s^j \right) \right] \\
\text{s.t.} \quad &\sum_{s \geq t} \beta^{s-t} u(c_s^j - v(\ell_s^j)) \geq \sum_{s \geq t} \beta^{s-t} u(c_s^j - v(\ell_s^j)) \\
\iff &\{c_s^j - v(\ell_s^j), \ell_s^j\}_{s \geq t} \in \arg\min_{\{c_s^j, \ell_s^j\}_{s \geq t} \text{ feasible}} \sum_{s \geq t} R^{(s-t)} \left[ \sum_{j \in I} (c_s^j + v(\ell_s^j)) - F \left( \sum_{j \in I} \ell_s^j, \sum_{j \in I} \ell_s^j \right) \right] \\
\text{s.t.} \quad &\sum_{s \geq t} \beta^{s-t} u(\tilde{c_s}^j) \geq \sum_{s \geq t} \beta^{s-t} u(c_s^j - v(\ell_s^j)) \\
\iff &\{\ell_s^j\}_{s \geq t} \in \arg\max_{\{\ell_s^j\}_{s \geq t} \text{ feasible}} \sum_{s \geq t} R^{(s-t)} \left[ F \left( \sum_{j \in I} \ell_s^j, \sum_{j \in I} \ell_s^j \right) - \sum_{j \in I} v(\ell_s^j) \right] \quad (A1) \\
\{c_s^j - v(\ell_s^j)\}_{s \geq t} \in \arg\min_{\{\tilde{c_s}^j\}_{s \geq t}} \sum_{s \geq t} R^{(s-t)} \sum_{j \in I} \tilde{c_s}^j \\
\text{s.t.} \quad &\sum_{s \geq t} \beta^{s-t} u(\tilde{c_s}^j) \geq \sum_{s \geq t} \beta^{s-t} u(c_s^j - v(\ell_s^j))
\end{align*}$$
\[
\{\ell_j^i\}_{j \in I} \in \arg\max_{\text{feasible} \{\ell_j^i\}_{j \in I}} \sum_{s \geq t} R^{-(s-t)} \left[ F \left( \sum_{j \in I} \hat{\ell}_j^s, \sum_{j \in I} \hat{\ell}_j^s \right) - \sum_{j \in I} v(\hat{\ell}_j^s) \right]
\]

\[
\{c_j^i\}_{j \geq t} \in \arg\min_{\{c_j^i\}_{j \geq t}} \sum_{s \geq t} R^{-(s-t)} \sum_{j \in I} c_j^s \\
\text{s.t.} \quad \sum_{s \geq t} \beta^{s-t} u(c_j^s - v(\hat{\ell}_j^s)) \geq \sum_{s \geq t} \beta^{s-t} u(c_j^i - v(\hat{\ell}_j^i))
\]

\[
\{\ell_j^i\}_{j \geq t} \in \arg\max_{\text{feasible} \{\ell_j^i\}_{j \geq t}} \sum_{s \geq t} R^{-(s-t)} \left[ F \left( \sum_{j \in I} \hat{\ell}_j^s, \sum_{j \in I} \hat{\ell}_j^s \right) - \sum_{j \in I} v(\hat{\ell}_j^s) \right],
\]

where “feasible” refers to the first bullet in the conditions for Pareto efficiency above. The second part of the third and fourth sets of conditions hold for all \(j \in I\). The final equivalence follows from household optimization with \(u\) strictly increasing, using that \(\{c_j^i, \ell_j^i\}_{j \geq t}\) the path of consumption and labor supply in an equilibrium.

\[\square\]

**Lemma 2.** Fix a history \(h^t\) and a household \(i\). The profile \(\{c_j^i, \ell_j^i\}_{j \geq t}\) satisfies household optimization for \(i\) starting from \(h^t\) if and only if

\[
\{c_j^i\}_{j \geq t} \in \arg\max_{\{c_j^i\}_{j \geq t}} \sum_{s \geq t} \beta^{-(s-t)} u(\hat{c}_j^i - v(\hat{\ell}_j^i)) \\
\text{s.t.} \quad d^i(h^t) + \sum_{s \geq t} R^{-(s-t)} \left( \hat{c}_j^i - \omega_j^i(\hat{\ell}_j^i; h^t) \cdot \hat{\ell}_j^i - T_j^i(h^t) \right) \leq 0 \quad (A2)
\]

\[
\{\ell_j^i\}_{j \geq t} \in \arg\max_{\text{feasible} \{\ell_j^i\}_{j \geq t}} \sum_{s \geq t} R^{-(s-t)} \left( \omega_j^i(\hat{\ell}_j^i; h^t) \cdot \hat{\ell}_j^i - v(\hat{\ell}_j^i) \right).
\]

**Proof.** We use the GHH structure of household preferences to rearrange the condition for
A.2 Proof of Theorem 1

Before we begin, a few pieces of notation:

- For any history of factor supply adjustments \( h^t = (h_{t-1}, ..., h_0) \), let \( \{\hat{\ell}^i_s(h^t)\}_{s \leq t} \) be the corresponding history of factor supply, i.e. for all \( j \in \mathcal{I} \), \( \hat{\ell}^j_s(h^t) = \bar{P}_0 \), and for \( s = 1, ..., t \), \( \hat{\ell}^j_s(h^t) = 1_{j=i_{s-1}} h_{s-1} + 1_{j \neq i_{s-1}} \hat{\ell}_{s-1}^j(h^t) \).
- Similarly, for any feasible history of factor supply \( \{\ell^i_s\}_{s \leq t} \), let \( \hat{h}^t(\{\ell^i_s\}_{s \leq t}) = (\ell^{i_{t-1}}, ..., \ell^{i_0}) \) be the corresponding history of factor supply adjustments.
- For any history \( h^t \), let \( \Pi(h^t) \) denote the set of all feasible factor supply paths that pass through \( h^t \), i.e.

\[
\Pi(h^t) = \left\{ \{\ell^i_s\}_{s \leq t} \mid \{\ell^i_s\}_{s \leq t} = \{\hat{\ell}^i_s(h^t)\}_{s \leq t}, \text{ and } t' > t, \{\ell^{i_{t'}}_{t'}\}_{s \leq t'} \in \Gamma \left( \{\ell^{i_{s}}_{s \leq t'}\}_{s \leq t'} \right) \right\},
\]

where \( \Gamma \left( \{\ell^{i_{s'}}_{s<s'}\}_{s<s'} \right) = \left\{ \{\hat{\ell}^{i_{j}}_{j}\}_{j \in \mathcal{I}} \mid \forall j \in \mathcal{I}, \hat{\ell}^{i_{j}} = \ell^{i_{j-1}} \text{ unless } j = i_{s'-1} \right\} \).

Fix a history \( h^t \). In order to establish that the equilibrium path following \( h^t \) is Pareto-optimal:

\[
\{c^i_s, \ell^i_s\}_{s \geq t} \in \arg\max_{\text{feasible } \{c^i_s, \ell^i_s\}_{s \geq t}} \sum_{s \geq t} \beta^{-(s-t)} u \left( c^i_s - v(\hat{\ell}^i_s) \right) \\
\text{s.t. } d^i(h^t) + \sum_{s \geq t} R^{-(s-t)} \left( c^i_s - \omega^i_s(\hat{\ell}^i_s; h^t) \cdot \hat{\ell}^i_s - T^i_s(h^t) \right) \leq 0
\]

\[\iff\]

\[
\{c^i_s - v(\hat{\ell}^i_s), \ell^i_s\}_{s \geq t} \in \arg\max_{\text{feasible } \{c^i_s, \ell^i_s\}_{s \geq t}} \sum_{s \geq t} \beta^{-(s-t)} u(\hat{C}^i_s) \\
\text{s.t. } d^i(h^t) + \sum_{s \geq t} R^{-(s-t)} \left( \hat{C}^i_s + v(\hat{\ell}^i_s) - \omega^i_s(\hat{\ell}^i_s; h^t) \cdot \hat{\ell}^i_s - T^i_s(h^t) \right) \leq 0
\]

\[\iff\]

\[
\{c^i_s - v(\hat{\ell}^i_s)\}_{s \geq t} \in \arg\max_{\{\hat{C}^i_s\}_{s \geq t}} \sum_{s \geq t} \beta^{-(s-t)} u(\hat{C}^i_s) \\
\text{s.t. } d^i(h^t) + \sum_{s \geq t} R^{-(s-t)} \left( \hat{C}^i_s + v(\hat{\ell}^i_s) - \omega^i_s(\hat{\ell}^i_s; h^t) \cdot \hat{\ell}^i_s - T^i_s(h^t) \right) \leq 0
\]

\[
\{\ell^i_s\}_{s \geq t} \in \arg\min_{\text{feasible } \{\ell^i_s\}_{s \geq t}} \sum_{s \geq t} R^{-(s-t)} \left( v(\hat{\ell}^i_s) - \omega^i_s(\hat{\ell}^i_s; h^t) \cdot \hat{\ell}^i_s \right).
\]

(A3)

where the final equivalent statement uses that \( u \) is non-satiated. \( \square \)
efficient, it suffices by Lemma 1 to show that
\[
\{\ell^j_s(h^t)\}_{s \geq t} \in \arg \max_{\{\ell^j_s\}_{s \geq t} \in \Pi(h^t)} W\left(\{\ell^j_s\}_{s \geq t}\right) = \arg \max_{\{\ell^j_s\}_{s \geq t} \in \Pi(h^t)} \sum_{s \geq t} R^{-s-t} \left[ F \left( \sum_{j \in I} \ell^j_s, \sum_{j \in I} \ell^j_{s'} \right) - \sum_{j \in I} v(\ell^j_s) \right].
\]
(A5)

By Theorems 4.3 and 4.5 of Stokey et al. [1989] the boundedness assumption in Footnote 1, it suffices to show there exists a function \( \hat{V} \) that, for all histories \( h'' \) that can feasibly follow \( h' \), satisfies
\[
\hat{V} \left( \{\ell^j_s(h'')\}_{s \leq t} \right), \ell_{t'+1}^j(h'') \\
\in \max_{\ell \in \Gamma(\{\ell^j_s(h'')\}_{s \leq t})} \left[ F \left( \sum_{j \in I} \ell^j_t(h''), \sum_{j \in I} \ell^j_{t'}(h'') \right) - \sum_{j \in I} v(\ell^j_t(h'')) \right] + R^{-1} \hat{V} \left( \{\ell^j_s(h'')\}_{s \leq t'} \right)
\]
(A6)
or equivalently,
\[
\hat{V} \left( \{\ell^j_s(h'')\}_{s \leq t} \right), \ell^j_{t'+1}(h'') \\
= \max_{\ell \in \Lambda} \left[ F \left( \sum_{j \in I} \ell^j_t(h''), \sum_{j \in I} \ell^j_{t'}(h'') \right) - \sum_{j \in I} v(\ell^j_t(h'')) \right] + R^{-1} \hat{V} \left( \{\ell^j_s(h'')\}_{s \leq t'} \right).
\]
(A7)

To this end, for all feasible histories of factor supply \( \{\ell^j_s\}_{s \leq t} \), define \( \hat{V}(\{\ell^j_s\}_{s \leq t}) \) as the NPV of future aggregate output less factor supply disutility on the equilibrium path:
\[
\hat{V}(\{\ell^j_s\}_{s \leq t}) = W\left( \{\ell^j_t(h'') \} \right).
\]
(A8)

By (a) household optimality, (b) the definition of super-Pigouvian policy, and (c) the definition of \( W(\cdot) \), we have that for any history \( h'' = (h_{t'-1}, ..., h_0) \) that can feasibly follow
Moreover, the definition of \( W(\cdot) \) implies that the right-hand side of (A9) evaluated at \( \ell^{t'} = \ell^{t'}_{t+1}(h^{t'}) \) is equal to \( W(\{\ell^{t'}_{i}(h^{t'})\}_{i \in I}^{t'}) \). So (A7) holds, completing the proof.

### A.3 Proof of Theorem 2

Suppose that the functions \( \{c^{i}, \ell^{i}\}_{i \in I} \) satisfy (7) and generate paths \( \{c^{i}_{s}(h^{t}), \ell^{i}_{s}(h^{t})\}_{s \geq t} \) starting from any history \( h^{t} \) that are Pareto efficient taking as given the labor allocation at \( t \) in that history and aggregate debt \( D(h^{t}) \) accumulated along that history.

Starting from any history \( h^{t} \), (a) the fact that \( u \) is non-satiated and (b) Lemma 1 imply that

\[
\{\ell^{i}_{s}(h^{t})\}_{s \geq t} \in \arg \max \limits_{\{\ell^{i}_{s}\}_{s \geq t} \text{ feasible } h^{t} \in I} W(\{\ell^{i}_{s}\}_{s \geq t})
\]

\[
\forall i \in I, \quad \{c^{i}_{s}(h^{t})\}_{s \geq t} \in \arg \max \limits_{\{c^{i}_{s}\}_{s \geq t} \text{ s.t. } \sum_{s \geq t} R^{-s(t)} c^{i}_{s} \leq \sum_{s \geq t} R^{-s(t)} c^{i}_{s}(h^{t})} \sum_{s \geq t} \beta^{s-t} u(c^{i}_{s} - v(\ell^{i}_{s}(h^{t})))
\]

Note that the above condition for factor supply in particular implies that for all histories \( h^{t} = (h_{t-1}, ..., h_{0}) \),

\[
\ell^{i}_{t-1}(h^{t}) \in \arg \max \limits_{\ell \in A} W(\{\ell^{i}_{s}(\ell, h_{t-2}, ..., h_{0})\}_{s \geq t})
\]

We now construct the equilibrium as follows: Let consumption and factor supply be given
by \( \{c^i, \ell^i\}_{i \in I} \). For any history \( h^t \) and any \( s \geq t \),

\[
\begin{align*}
L_s(h^t) &= \sum_{i \in I} \ell^i_s(h^t) \\
Y_s(h^t) &= F\left( L_s(h^t), L_s(h^t) \right) \\
w_s(h^t) &= F_L\left( L_s(h^t), L_s(h^t) \right)
\end{align*}
\]

\[
\tau^i_s(h; h^t) \cdot \ell = \begin{cases} 
-W \left( \left\{ \ell_s^i \cap \{\ell^\ell^t_{u+1}(h^t)\}_{u^t=t-\ldots-s-1} \right\} \right)_{j \in I} \\
+ \sum_{t' = s+1}^{t_{\text{next}}(i, s)} R^{-t'(s-1)} \left( w_{t'}(h^t) \cdot \ell - v(\ell) \right) & \text{if } i = i_s, \\
0 & \text{otherwise}
\end{cases}
\]

\[
\omega^i_s(h; h^t) \cdot \ell = w_s(h^t) \cdot \ell - \tau^i_s(h; h^t) \cdot \ell
\]

\[
T^i_s(h^t) = \begin{cases} 
\sum_{t' = t}^{t_{\text{next}}(i, s)} R^{-t'(s-1)} \left[ c^i_s(h^t) - \omega^i_s(h^t) \cdot \ell_s^i(h^t) \right] + \delta^i(h^t) & \text{if } s = t \\
0 & \text{otherwise},
\end{cases}
\]

(A12)

where \( t_{\text{next}}(i, s) \) is the next period after \( s \) with \( i_s = i_{t_{\text{next}}(i, s)} \) (and \( T \) if no such period exists), and where note that \( T^i_s(h^t) \) is defined recursively, since \( \delta^i(h^t) \) depends on transfers at times before \( t \). Note that \( \tau^i_s(h; h^t) \) is constructed so that for any history \( h^t \) and any time \( t' \geq t \) with \( i = i' \),

\[
\sum_{s = t' + 1}^{t_{\text{next}}(i, s)} R^{-t'(s-t'+1)} \left[ \left( w_s(h^t) - \tau^i_s(h; h^t) \right) \cdot \ell - v(\ell) \right]
= W \left( \left( \{\ell_s^i(\ell \cap \{\ell^\ell^t_{u+1}(h^t)\}_{u^t=t-\ldots-s-1} \} \right)_{j \in I} \right)_{s \geq t'.}
\]

(A13)

As the equilibrium consumption and factor supply profile is Pareto efficient starting from any history by construction, it suffices to verify that this profile is a super-Pigouvian equilibrium, i.e. (7) and (2)–(6) hold. (7) holds for consumption and factor supply by assumption, and therefore holds for all other equilibrium functions because they are constructed from the consumption and factor supply functions.

Of the conditions (2)–(6), all are immediate from their construction and—in the case of goods market clearing—the fact that for a profile to be Pareto efficient from any history it must satisfy goods market clearing from any history, except for household optimality. By Lemma 2, it suffices to show, for every history \( h^t \) and household \( i \) that

\[
\{c^i_s(h^t)\}_{s \geq t} \in \arg \min_{\{\ell^i_s\}_{s \geq t}} \sum_{s \geq t} R^{-t} u(c^i_s - v(\ell^i_s(h^t)))
\]

s.t. \( \sum_{s \geq t} R^{-t} c^i_s \leq \sum_{s \geq t} R^{-t} \left[ \omega^i_s(\ell^i_s(h^t); h^t) \cdot \ell^i_s(h^t) + T^i_s(h^t) \right] - \delta^i(h^t) \)

\[
\{\ell^i_s(h^t)\}_{s \geq t} \in \arg \min_{\text{feasible } \{\ell^i_s\}_{s \geq t}} \sum_{s \geq t} R^{-t} \left( \omega^i_s(\ell^i_s(h^t); \ell^i_s) - v(\ell^i_s) \right)
\]

(A14)
To see that the condition for consumption is true, note that by the definition of transfers, the budget constraint can be replaced by $\sum_{s \geq t} R^{-s-t} \ell^i_s \leq \sum_{s \geq 1} R^{-s-t} C_s^j(h^t)$. The fact that consumption satisfies this condition then follows immediately from the Pareto efficiency of the profile with which we began the proof. Next, consider the condition for labor supply. Note that it is equivalent to the condition that for all times $t' \geq t$ at which $i = i_{t'}$,

$$
\ell^i_{t'+1}(h^t) \in \arg\min_{\ell \in \mathcal{A}} \sum_{s=t+1}^{t_{\text{next}}(i,t')} R^{-s-(t'+1)} \left( \omega^j_s(\ell; h^t) \cdot \ell - v(\ell) \right)
$$

$$
\iff
\ell^i_{t'+1} \left( \{ \ell^i_{t+1}(h^t) \}_{t'=t+1}^{h^t} \right) \in \arg\min_{\ell \in \mathcal{A}} W \left( \{ \ell^i_s(\ell \land \ell^i_{t'+1}(h^t)) \}_{s=t+1}^{h^t} \right)
$$

(A15)

where the equivalence follows from—on the LHS—the fact that the profile we began with satisfies (7) and—on the RHS—(A13). As desired, the last condition holds by (A11).

### A.4 Proof of Proposition 2

Below, I state a full version of the proposition that defines the $\delta_{SP}$ term used in the main text. To do so, I introduce the following notation: Fix a history $h^{t>0} = (h_{t-1}, ..., h_0)$. First, denote by $O^{i_{t-1}}(\Delta \ell, h^t)$ $i_{t-1}$'s utility loss—in NPV consumption terms, starting from $t$—from misoptimizing along the path where prices and transfers follow $h^t$ but $i_{t-1}$'s initial labor supply is $h_{t-1} + \Delta \ell$ that results from $i_{t-1}$ playing according to her strategy that is optimal if prices and transfers follow her deviation. Second, denote by $I^{i_{t-1}}(\Delta \ell, h^t)$ the internality bias—again, in NPC consumption units starting from $h^t$—that $i_{t-1}$ has toward her equilibrium behavior according to the deviated-by-$\Delta \ell$ path following $h^t$ because she believes that prices and transfers evolve as they do on the equilibrium path following $h^t$ rather than on the deviated-by-$\Delta \ell$ path.\(^{27}\)

Importantly, note that both of these terms disappear in any limit where $i_{t-1}$ has no market power, i.e. cannot influence prices or transfers (except through the quantity-dependence of marginal taxes). For $O^{i_{t-1}}$, this is because if $i_{t-1}$'s behavior has no effect on prices, then her optimal behavior following $h^t$ and $(h_{t-1} + \Delta \ell, h_{t-2}, ..., h_0)$ only differs due to the change in initial condition. For $I^{i_{t-1}}$, this is because $i_{t-1}$ has no internality if her incorrect belief that she does not affect prices or transfers is, in fact, true.

**Proposition 2’.** An equilibrium is super-Pigouvian if and only if for all histories $h^{t>0} = \ldots$
\[(h_{t-1}, ..., h_0)\) and all alternative histories \(h' = (h_{t-1} + \Delta \ell, ..., h_0)\) with \(h_{t-1} + \Delta \ell \in \mathcal{A},\)

\[
\sum_{j \neq i_{t-1}} c^j \left( \left\{ c^j_s(h'|t), \ell^j_s(h'|t) \right\}_{s \geq t}, \left\{ c^j_s(h'), \ell^j_s(h') \right\}_{s \geq t} \right) - O^i(\Delta \ell, h) - I^i(\Delta \ell, h) = 0. \quad (A17)
\]

Fix a history \(h^t = (h_0, ..., h_0)\) and for notational convenience, let \(i \equiv i_{t-1}.\) Fix some \(\Delta \ell\) such that \(h_{t-1} + \Delta \ell \in \mathcal{A},\) and let \(h' = (h_{t-1} + \Delta \ell, \ldots, h_0).\) Finally, let \(t'\) denote the next time after \(t \to t'\) at which \(i = i_{t'},\) or \(|\mathcal{T}|\) if no such time exists. We then have

\[
\sum_{j \neq i} c^j \left( \left\{ c^j_s(h'|t), \ell^j_s(h'|t) \right\}_{s \geq t}, \left\{ c^j_s(h'), \ell^j_s(h') \right\}_{s \geq t} \right) = \sum_{s \geq t} R^{-s-t} \sum_{j \neq i} \left[ \left( c^j_s(h'|t) - v(\ell^j_s(h'|t)) \right) \right.
\]

\[- \left( c^j_s(h') - v(\ell^j_s(h')) \right) \]

\[
= \sum_{s \geq t} R^{-s-t} \sum_{j \neq i} \left[ \left( \omega^j_s(\ell^j_s(h'|t)\cdot \ell^j_s(h'|t) + T^j_s(h'|t) - v(\ell^j_s(h'|t)) \right) \right.
\]

\[- \left( \omega^j_s(\ell^j_s(h')\cdot \ell^j_s(h') + T^j_s(h') - v(\ell^j_s(h')) \right) \]

\[
= \sum_{s \geq t} R^{-s-t} \sum_{j \neq i} \left[ \left( w^j_s(h'|t)\cdot \ell^j_s(h'|t) - v(\ell^j_s(h'|t)) \right) - w^j_s(h')\cdot \ell^j_s(h') + v(\ell^j_s(h')) \right]
\]

\[-\tau^j_s(\ell^j_s(h'|t); h'|t)\cdot \ell^j_s(h'|t) + T^j_s(h'|t) + \tau^j_s(\ell^j_s(h'); h')\cdot \ell^j_s(h') - T^j_s(h') \right] \]

\[
= \sum_{s \geq t} R^{-s-t} \left[ F(\mathbf{L}_s(h'|t), \mathbf{L}_s(h')) = F(\mathbf{L}_s(h'), \mathbf{L}_s(h')) \right]
\]

\[-\sum_{j \neq i} \left( v(\ell^j_s(h'|t)) - v(\ell^j_s(h')) \right) \]

\[-\sum_{j \neq i} \left( v(\ell^j_s(h'|t)) - v(\ell^j_s(h')) \right) \]

\[
+ \tau^j_s(\ell^j_s(h'|t); h'|t)\cdot \ell^j_s(h'|t) - T^j_s(h'|t) - \tau^j_s(\ell^j_s(h'); h')\cdot \ell^j_s(h') + T^j_s(h') \right] \]

\[
= \sum_{s \geq t} R^{-s-t} \left[ F(\mathbf{L}_s(h'|t), \mathbf{L}_s(h')) \right]
\]

\[-\sum_{j \neq i} \left( v(\ell^j_s(h'|t)) - v(\ell^j_s(h')) \right) \]

\[-\sum_{j \neq i} \left( v(\ell^j_s(h'|t)) - v(\ell^j_s(h')) \right) \]

\[
+ \tau^j_s(\ell^j_s(h'|t); h'|t)\cdot \ell^j_s(h'|t) - T^j_s(h'|t) - \tau^j_s(\ell^j_s(h'); h')\cdot \ell^j_s(h') + T^j_s(h') \right] \]

\[
= I^i(\Delta \ell, h') + W \left( \left\{ \ell^j_s((h_{t-1} + \Delta \ell, h_{t-2}, ..., h_0)) \right\}_{s \geq t} \right) - W \left( \left\{ \ell^j_s(h') \right\}_{s \geq t} \right)
\]

\[
+ \sum_{s \geq t} R^{-s-t} \left[ -\omega^j_s(\ell^j_s(h'|t)\cdot \ell^j_s(h'|t) - T^j_s(h'|t) + v(\ell^j_s(h'|t)) \right]
\]

\[-\sum_{j \neq i} \left( v(\ell^j_s(h'|t)) - v(\ell^j_s(h')) \right) \]

\[-\sum_{j \neq i} \left( v(\ell^j_s(h'|t)) - v(\ell^j_s(h')) \right) \]

\[
+ \omega^j_s(\ell^j_s(h'|t); h')\cdot \ell^j_s(h') + T^j_s(h') - v(\ell^j_s(h')) \right] \]

\[
= I^i(\Delta \ell, h') + O^i(\Delta \ell, h') + W \left( \left\{ \ell^j_s((h_{t-1} + \Delta \ell, h_{t-2}, ..., h_0)) \right\}_{s \geq t} \right) - W \left( \left\{ \ell^j_s(h') \right\}_{s \geq t} \right)
\]

\[
+ \sum_{s \geq t} R^{-s-t} \left[ -\omega^j_s(\ell^j_s(h'|t) + 1_{s \leq t'} \Delta \ell)\cdot \ell^j_s(h') + v(\ell^j_s(h') + 1_{s \leq t'} \Delta \ell) \right]
\]
\[
\begin{align*}
&+ \omega_i'(\ell^i_s(h^t); h^t) \cdot \ell^i_s(h^t) + T^i_s(h^t) - v(\ell^i_s(h^t)) \\
&= I^i(\Delta \ell, h^t) + O^i(\Delta \ell, h^t) + W\left(\{\ell^i_s((h_{t-1} + \Delta \ell, h_{t-2}, \ldots, h_0))\}_{s \geq t}\right) - W\left(\{\ell^i_s(h^t)\}_{s \geq t}\right) \\
&- \Delta \tilde{U}^i(\Delta \ell, h^t),
\end{align*}
\]

Above, the first equality is by the definition of \( e^j \). The second equality is by the household budget constraint. The third and fifth inequalities are by the equilibrium condition relating pre- and post-tax wages. The fourth inequality is by (a) the fact that production is constant returns to scale and (b) the government budget constraint (which recall is implied by Walras’ law) and the fact that \( h^t \) and \( h^t \) are the same up until \( t^+(h) \). The sixth equality is by the definitions of \( I^i \) and \( W^i \). The seventh equality is by the definition of \( O^i \) and the observation that—conditional on playing \( h_{t-1} + \Delta \ell \) at \( t - \ell^i_s(h^t) + 1_{s \leq t} \Delta \ell \) is optimal behavior given prices following \( \tilde{\ell}^t \), since by household optimality \( \ell^i_s(h^t) \) is optimal ignoring the constraint of playing \( \ell^i_s \) at \( t \). The seventh equality is by the definition of \( \Delta \tilde{U}^i \).

Moving \( I^i(\Delta \ell, h^t) \) and \( O^i(\Delta \ell, h^t) \) to the left-hand side reveals that (A17) holds (i.e. the LHS is zero) for all histories \( h^t \) if and only if (16) holds (i.e. the RHS is zero) for all histories \( h^t \).
B Analysis of quantitative model

B.1 Characterization of sectoral laboral labor supply

Each unit of labor \(m\) is assigned to the traditional sector if and only if \(w_t^T a(m) > w_t^I\). So

\[
L_t^I = M \mathbb{P}[a \leq w_t^I/w_t^T] = M \left( \frac{w_t^I}{\bar{a} w_t^T} \right)^\varepsilon, \tag{A19}
\]

provided \(w_t^I/w_t^T \leq \bar{a}\). Similarly,

\[
L_t^T = M \int_{w_t^I/w_t^T}^{\bar{a}} a \frac{d}{da} \mathbb{P}[\bar{a} \leq a] da = M \int_{w_t^I/w_t^T}^{\bar{a}} a \varepsilon \frac{a^{\varepsilon-1}}{\bar{a}^{\varepsilon}} da = M \frac{\varepsilon}{\varepsilon + 1} \frac{1}{\bar{a}^{1+\varepsilon}} \left( \frac{w_t^I}{w_t^T} \right)^{1+\varepsilon}. \tag{A20}
\]

B.2 Characterization of wages, profits, and output

I now characterize the equilibrium of the quantitative model of Section 6 at any period \(t\), given some fixed \(N_t\) and \(M\).

I start with the pricing conditions for variety and final goods producers, respectively. The former sell at a common price \(p_t\)

\[
p_t = \frac{\eta}{\eta-1} c_t = \frac{\eta}{\eta-1} \left( \frac{w_t^I}{\alpha} \right)^\alpha \left( \frac{1}{1-\alpha} \right)^{1-\alpha}, \tag{A21}
\]

whereas the latter’s price-normalization condition implies

\[
1 = \frac{N_t^{1-\eta}}{z_t} p_t \implies p_t = z_t N_t^{\frac{1}{1-\eta}} \tag{A22}
\]

Plugging in for \(p_t\) in the variety pricing equation gives an expression for wages in the industrial sector:

\[
w_t^I = \left( \frac{\eta-1}{\eta} \alpha^\alpha (1-\alpha)^{1-\alpha} \right)^{\frac{1}{\alpha}} \frac{1}{p_t^{1/\alpha}} = \left( \frac{\eta-1}{\eta} \alpha^\alpha (1-\alpha)^{1-\alpha} \right)^{\frac{1}{\alpha}} N_t^{\frac{1}{1-\eta-1}} z_t^{\frac{1}{\alpha}}. \tag{A23}
\]

Note that this expression and \(w_t^T = 1\) imply that the relative wage in industry is bound below \((\frac{\eta-1}{\eta} \alpha^\alpha (1-\alpha)^{1-\alpha})^{\frac{1}{\alpha}} (N + I)^{\frac{1}{1-\eta+1}} e^{\frac{1}{2} z} \) if \(z_t \leq \bar{z}\). In order to guarantee that \(w_t^I \leq \bar{a} w_t^T\)—as discussed in the main text—1 assign \(\bar{z}\) a maximum value in my simulation of the model and take \(\bar{a}\) greater than the expression above.

Next, consider firms labor and input demands:

\[
x_t = (1-\alpha) c_t q_t = (1-\alpha) \frac{\eta}{\eta-1} p_t q_t, \quad l_t = \alpha \frac{\eta-1}{\eta} N_t^{\frac{1}{1-\eta}} p_t q_t \tag{A24}
\]

Moreover, note that, as \(\chi_t = 1\) by perfect competition, the labor supply to the industrial
we can solve for output per firm

sector takes the form

\[ L_t^I = M \bar{a}^{-\varepsilon} (w_t^I)^{\varepsilon}. \]  

(A25)

Combining the above expressions for labor supply, labor demand, and the industrial wage, we can solve for output per firm \( q_t \):

\[
N_t \alpha \frac{\eta - 1}{\eta} \frac{p_t}{w_t^I} q_t = N_t t_t = M \bar{a}^{-\varepsilon} (w_t^I)^{\varepsilon} \\
\implies q_t = \frac{1}{\alpha} \frac{\eta}{\eta - 1} \frac{(w_t^I)^{1+\varepsilon}}{p_t} M \bar{a}^{-\varepsilon} \\
= M \bar{a}^{-\varepsilon} \frac{1}{\alpha} \frac{\eta}{\eta - 1} \left( \frac{\eta - 1}{\eta} \alpha^{1 - (1 - \alpha)} \right)^{\frac{1+\varepsilon}{\alpha}} \frac{1 + \varepsilon}{\alpha} \frac{1}{z_t} \frac{1 + \varepsilon}{\alpha} \frac{1}{N_t} - \frac{\eta}{\eta - 1} \\
= M \bar{a}^{-\varepsilon} \alpha^{\varepsilon} (1 - \alpha)^{\frac{1-\alpha}{\alpha} (1+\varepsilon)} \left( \frac{\eta - 1}{\eta} \right)^{\frac{1+\varepsilon}{\alpha}} \frac{1 + \varepsilon}{\alpha} \frac{1}{z_t} \frac{1 + \varepsilon}{\alpha} \frac{1}{N_t} \frac{1}{N_t^{\alpha (\eta - 1)}}.
\]

We can now compute variety firm profits

\[
\pi_t = \left( \frac{\eta}{\eta - 1} - 1 \right) c_t q_t - f \\
= \left( \frac{\eta}{\eta - 1} - 1 \right) \frac{\eta - 1}{\eta} p_t q_t - f \\
= \frac{1}{\eta} \cdot \sum_{t} N_t^{\frac{\eta - 1}{\eta - 1}} \cdot M \bar{a}^{-\varepsilon} (1 - \alpha)^{\frac{1-\alpha}{\alpha} (1+\varepsilon)} \left( \frac{\eta - 1}{\eta} \right)^{\frac{1+\varepsilon}{\alpha}} \frac{1 + \varepsilon}{\alpha} \frac{1}{z_t} \frac{1 + \varepsilon}{\alpha} \frac{1}{N_t} \frac{1}{N_t^{\alpha (\eta - 1)}} - f. \\
= \bar{a}^{-\varepsilon} \alpha^{\varepsilon} (1 - \alpha)^{\frac{1-\alpha}{\alpha} (1+\varepsilon)} \eta^{\frac{1+\varepsilon}{\alpha}} \left( \frac{\eta - 1}{\eta} \right)^{\frac{1+\varepsilon}{\alpha}} \frac{1 + \varepsilon}{\alpha} \frac{1}{z_t} \frac{1 + \varepsilon}{\alpha} \frac{1}{N_t} \frac{1}{N_t^{\alpha (\eta - 1)}} - f.
\]

(B.3) Characterization of aggregate output and Pigouvian policy

I also compute aggregate net output for later use.

\[
Y_t = Q_t - N_t x_t - N_t f + L_t^T = N_t (p_t q_t - x_t) - N_t f + L_t^T \\
= N_t \left( p_t q_t - (1 - \alpha) \frac{\eta - 1}{\eta} p_t q_t \right) - N_t f + L_t^T \\
= \left( 1 - (1 - \alpha) \frac{\eta - 1}{\eta} \right) p_t q_t N_t - N_t f + \varepsilon \frac{\varepsilon}{\varepsilon + 1} M \left( \bar{a} - \frac{(w_t^I)^{1+\varepsilon}}{\bar{a}^{\varepsilon}} \right) \\
= \left( 1 - (1 - \alpha) \frac{\eta - 1}{\eta} \right) z_t N_t^{\frac{\eta - 1}{\eta}} \bar{a}^{-\varepsilon} \alpha^{\varepsilon} (1 - \alpha)^{\frac{1-\alpha}{\alpha} (1+\varepsilon)} \left( \frac{\eta - 1}{\eta} \right)^{\frac{1+\varepsilon}{\alpha}} \frac{1 + \varepsilon}{\alpha} \frac{1}{z_t} \frac{1 + \varepsilon}{\alpha} \frac{1}{N_t} \frac{1}{N_t^{\alpha (\eta - 1)}} N_t - N_t f \\
+ \varepsilon \frac{\varepsilon}{\varepsilon + 1} M \bar{a} - \varepsilon \frac{\varepsilon}{\varepsilon + 1} M \bar{a}^{-\varepsilon} \left( \frac{\eta - 1}{\eta} \alpha^{\varepsilon} (1 - \alpha)^{\frac{1-\alpha}{\alpha} (1+\varepsilon)} \left( \frac{\eta - 1}{\eta} \right)^{\frac{1+\varepsilon}{\alpha}} \frac{1 + \varepsilon}{\alpha} \frac{1}{z_t} \frac{1 + \varepsilon}{\alpha} \frac{1}{N_t} \frac{1}{N_t^{\alpha (\eta - 1)}} \right) \\
= M \bar{a}^{-\varepsilon} \left( 1 - (1 - \alpha) \frac{\eta - 1}{\eta} \right) \alpha^{\varepsilon} (1 - \alpha)^{-\varepsilon} (1 - \alpha)^{\frac{1+\varepsilon}{\alpha} - 1} \left( \frac{\eta - 1}{\eta} \right)^{\frac{1+\varepsilon}{\alpha}} \frac{1 + \varepsilon}{\alpha} \frac{1}{z_t} \frac{1 + \varepsilon}{\alpha} \frac{1}{N_t} \frac{1}{N_t^{\alpha (\eta - 1)}} - N_t f + E_t.
\]
\[
\begin{align*}
+ \frac{\varepsilon}{\varepsilon + 1} M\bar{a} - \frac{\varepsilon}{\varepsilon + 1} M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{1+\varepsilon} \left( 1 - \frac{(1 - \alpha)\eta - 1}{\eta} \right)^{1+\frac{\varepsilon}{\alpha}} z_t^{\alpha} N_t^{\frac{1+\varepsilon}{\alpha}} f^{\frac{1+\varepsilon}{\alpha}} N_t^{\frac{1}{\alpha(\eta-1)}} \\
= M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{\varepsilon} \left( \frac{1}{1 - \alpha} \frac{\eta}{\eta - 1} - 1 \right) \left( 1 - \alpha \right) \frac{\eta - 1}{\eta} z_t N_t^{\frac{1}{\eta-1}} f^{\frac{1+\varepsilon}{\alpha}} N_t^{\frac{1}{\alpha(\eta-1)}} - N_t f \\
+ \frac{\varepsilon}{\varepsilon + 1} M\bar{a} - \frac{\varepsilon}{\varepsilon + 1} \frac{\alpha}{1 - \alpha} M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{\varepsilon} \left( 1 - \alpha \right) \frac{\eta - 1}{\eta} z_t N_t^{\frac{1}{\eta-1}} f^{\frac{1+\varepsilon}{\alpha}} N_t^{\frac{1}{\alpha(\eta-1)}} \\
= \left( \frac{1}{1 - \alpha} \left( \frac{\eta}{\eta - 1} - \alpha \frac{\varepsilon}{\varepsilon + 1} \right) - 1 \right) M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{\varepsilon} \left( 1 - \alpha \right) \frac{\eta - 1}{\eta} z_t N_t^{\frac{1}{\eta-1}} f^{\frac{1+\varepsilon}{\alpha}} N_t^{\frac{1}{\alpha(\eta-1)}} - N_t f + \frac{\varepsilon}{\varepsilon + 1} M\bar{a},
\end{align*}
\]

where here in the last line I have used that

\[
\frac{1}{1 - \alpha} \left( \frac{\eta}{\eta - 1} - \alpha \frac{\varepsilon}{\varepsilon + 1} \right) - 1 = \frac{\eta(\varepsilon + 1) - \alpha \varepsilon(\eta - 1) - (1 - \alpha)(\eta - 1)(\varepsilon + 1)}{(1 - \alpha)(\eta - 1)(\varepsilon + 1)}
\]

\[
= \frac{\eta(\varepsilon + 1) - (\eta - 1)(\alpha \varepsilon + \varepsilon - \alpha \varepsilon + 1 - \alpha)}{(1 - \alpha)(\eta - 1)(\varepsilon + 1)}
\]

\[
= \frac{\eta(\varepsilon + 1) - (\eta - 1)(\varepsilon + 1) + \alpha(\eta - 1)}{(1 - \alpha)(\eta - 1)(\varepsilon + 1)}
\]

\[
= \frac{(\varepsilon + 1) + \alpha(\eta - 1)}{(1 - \alpha)(\eta - 1)(\varepsilon + 1)}.
\]

We can also compute the marginal effect of entry on aggregate output and the difference between that and profits:

\[
\begin{align*}
\frac{dY_t}{dN_t} &= \frac{(\varepsilon + 1) + \alpha(\eta - 1)}{(1 - \alpha)(\eta - 1)^2} M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{\varepsilon} \left[ \left( 1 - \alpha \right) \frac{\eta - 1}{\eta} z_t N_t^{\frac{1}{\eta-1}} \right]^{\frac{1+\varepsilon}{\alpha}} N_t - f \\
\pi_t &= \frac{1}{(1 - \alpha)(\eta - 1)} M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{\varepsilon} \left[ \left( 1 - \alpha \right) \frac{\eta - 1}{\eta} z_t N_t^{\frac{1}{\eta-1}} \right]^{\frac{1+\varepsilon}{\alpha}} N_t - f \\
\pi_t &= \frac{\alpha(\eta - 1)}{(1 - \alpha)(\eta - 1)^2} M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{\varepsilon} \left[ \left( 1 - \alpha \right) \frac{\eta - 1}{\eta} z_t N_t^{\frac{1}{\eta-1}} \right]^{\frac{1+\varepsilon}{\alpha}} N_t - f \\
\frac{dY_t}{dN_t} - \pi_t &= \frac{(\varepsilon + 1)}{(1 - \alpha)(\eta - 1)^2} M\bar{a}^{-\varepsilon} \left( \frac{\alpha}{1 - \alpha} \right)^{\varepsilon} \left[ \left( 1 - \alpha \right) \frac{\eta - 1}{\eta} z_t N_t^{\frac{1}{\eta-1}} \right]^{\frac{1+\varepsilon}{\alpha}} N_t \\
= \frac{\varepsilon + 1}{(\varepsilon + 1) + \alpha(\eta - 1)(\pi_t + f)}.
\end{align*}
\]
Note this implies that—under the Pigouvian interpretation of the model—the optimal profits subsidy $-\tau_t$ is equal to

$$-\tau_t = \frac{dY_t}{dN_t} - \pi_t = \frac{\varepsilon + 1}{(\varepsilon + 1) + \alpha(\eta - 1)} \left(1 + \frac{f}{\pi_t}\right),$$  \hspace{1cm} (A31)

where $\pi_t$ is as above.

**B.4 Expressions used for calibration**

In this appendix I go through the mathematical steps required to back out various model parameters from moments of the data. I rely on expressions for various model relationships derived in Appendix B.2.

I set the elasticity of substitution across varieties to growth in match gross output output per worker and the number of firms—both of which are available in Lane [2022]—using the fact that

$$Q_t L_t = z_t N_t^{\frac{\eta}{\alpha}} \nu_t \equiv \frac{\eta}{\alpha - 1} \frac{(w_t^I)^{\eta + 1}}{N_t},$$  \hspace{1cm} (A32)

$$\eta = 1 + \frac{1}{\alpha} \frac{\Delta \log N_t + \Delta \log L_t}{\Delta \log Q_t}. \hspace{1cm} (A33)

I back out the mean of the productivity process $\mu$ to match total HCI output per worker, given
the number of firms, before the HCI drive. To do this, I express productivity \( z_t \) in terms of aggregate HCI output \( Q_t \), employment \( L_t^I \), and the number of establishments \( N_t \), all of which Lane [2022] observes.

\[
Q_t = N_t p_t q_t = N_t z_t N_t^{-\frac{1}{\eta}} M \hat{\alpha}^{-\varepsilon} \alpha (1 - \alpha)^{\frac{1}{\alpha}(1+\varepsilon)} \left( \eta - 1 \right)^{-\frac{1-\alpha}{\alpha}} z_t^{\frac{1-\alpha}{\alpha}} N_t^{\frac{1-\alpha+\varepsilon}{\alpha(\eta-1)}}
\]

\[
= L_t^I \left( \frac{\eta - 1}{\eta} N_t^{-\frac{1}{\eta}} \right)^{-\varepsilon} \alpha \varepsilon (1 - \alpha)^{\frac{1-\alpha}{\alpha}(1+\varepsilon)} \left( \eta - 1 \right)^{-\frac{1-\alpha+\varepsilon}{\alpha}} z_t^{\frac{1-\alpha+\varepsilon}{\alpha}} N_t^{\frac{1-\alpha+\varepsilon}{\alpha(\eta-1)}}
\]

\[
\left( \frac{Q_t}{L_t^I} \right)^{1+\varepsilon} = (1 - \alpha)^{\frac{1-\alpha}{\alpha}(1+\varepsilon)} \left( \frac{\eta - 1}{\eta} \right)^{\frac{1-\alpha}{\alpha}} z_t^{\frac{1-\alpha+\varepsilon}{\alpha}} N_t^{\frac{1-\alpha+\varepsilon}{\alpha(\eta-1)}}
\]

\[
\left( \frac{Q_t}{L_t^I} \right)^{\alpha} = \left( (1 - \alpha) \eta - 1 \right)^{\frac{1-\alpha}{\alpha}} z_t N_t^{-\frac{1}{\eta}}
\]

\[
\mu = \alpha \log(Q_t/L_t^I) - (1 - \alpha) \log \left( (1 - \alpha) \eta - 1 \right) - \frac{1}{\eta} \log N_t,
\]

where the last line follows from assuming that there is little enough variation in productivity that \( z_t = \exp(\mu) \).

I set \( f \) so that—under Pigouvian policy—entrant losses in the period before the HCI drive are equal to one half of entrant profits in the post-period. Our above expression for \( p_t q_t \), the expressions for \( \frac{dY}{dN} \) and \( \pi_t \) in Section B.2, and the assumption that productivity is the same before and after the HCI drive together imply that this is the case when

\[
\frac{dY_{pre}}{dN_{pre}} = -\frac{1}{2} \frac{dY_{post}}{dN_{post}},
\]

\[
\frac{dY_{pre}}{dN_{pre}} + f - f = -\frac{1}{2} \left( \frac{dY_{post}}{dN_{post}} + f - f \right)
\]

\[
\frac{\varepsilon + \alpha(\eta - 1)}{\alpha(\eta - 1)} (\pi_{pre} + f) - f = -\frac{1}{2} \left[ \frac{\varepsilon + \alpha(\eta - 1)}{\alpha(\eta - 1)} (\pi_{post} + f) - f \right]
\]

\[
\frac{\varepsilon + \alpha(\eta - 1)}{\alpha(\eta - 1)} \frac{1}{\eta} p_{pre} q_{pre} - f = -\frac{1}{2} \left[ \frac{\varepsilon + \alpha(\eta - 1)}{\alpha(\eta - 1)} \frac{1}{\eta} p_{post} q_{post} - f \right]
\]

\[
f = \frac{\varepsilon + \alpha(\eta - 1)}{\alpha(\eta - 1)} \frac{1}{2} \frac{1}{\eta} p_{post} q_{post} + \frac{1}{2} \frac{1}{\eta} p_{pre} q_{pre}
\]

\[
= \left( 1 + \frac{\varepsilon + 1}{\alpha(\eta - 1)} \right) \frac{1}{\eta} p_{pre} q_{pre} \left( \frac{N+I}{N} \right)^{\frac{1+\varepsilon}{\alpha(\eta-1)}} + 1
\]

\[
= \left( 1 + \frac{\varepsilon + 1}{\alpha(\eta - 1)} \right) \frac{1}{\eta} Q_{pre} \left( \frac{N+I}{N} \right)^{\frac{1+\varepsilon}{\alpha(\eta-1)}} + 1
\]

\[
\]
C Additional figures

Figure A1: Productivity cutoffs for entrepreneurial entry characterizing the highest- and lowest-industrializing equilibria of the calibrated model under Pigouvian policy, as well as the unique laissez-faire equilibrium.