Suppose a country anticipates that it may use trade as a bargaining chip in future geopolitical conflicts. How should it develop domestic industries and international trading relationships today in order to strengthen its hand tomorrow? We characterize the optimal capital accumulation and trade policies for this strategic (dis)integration. If the country can credibly threaten trade taxes tomorrow as punishment for geopolitical actions, then it abstains from capital subsidies today even though firms do not internalize their impact on the probability of conflict. Meanwhile, trade policy today seeks to influence foreign capital accumulation so as to make foreign prices more sensitive to trade tomorrow but not necessarily to increase foreign gains from trade. If the country’s ability to threaten trade taxes tomorrow is limited, capital subsidies today emerge as a second-best instrument. We show how such subsidies respond to realistic trade policy constraints, such as limited credibility and adherence to WTO rules.
1 Introduction

Recent geopolitical events have prompted countries around the world to reconsider their economic interdependence. In the wake of Russia’s invasion of Ukraine, Western nations imposed unprecedented sanctions on international trade. At the same time as preexisting trade relations with Russia made these sanctions possible, their interruption also caused significant economic distress in sanctioning countries. Many such countries have since pursued “de-risking,” “onshoring,” and “friend-shoring” policies to mitigate future geopolitical risks. Meanwhile, the United States passed sweeping legislation promoting its domestic semiconductor industry while at the same time banning exports of certain chips and preventing US engineers from working in some Chinese industries. Although some policymakers justified these actions on narrow economic and security grounds, others have offered explicitly geopolitical rationales about building strategic (in)dependence.

These events bring to the fore classic questions of economic statecraft. From a positive perspective, how can we understand the countries’ geopolitical incentives for economic policies beyond standard trade sanctions? From a normative perspective, how should such policies be designed to accomplish their aims without inflicting undue economic costs?

We shed new light on these questions by embedding a simple geopolitical game into an otherwise standard neoclassical model of international trade. In the model, countries have preferences over their economic welfare and each other’s geopolitical actions. The economic and geopolitical blocks of the model interact through economic threats—in the form of trade sanctions—that a country can make in order to influence foreign geopolitical behavior. We study how a country who anticipates making these threats in the future can use forward-looking policies during times of peace to prepare itself for conflict.

To highlight the economic mechanisms that underlie such policies, we focus on the case of two countries, “Home” and “Foreign,” who interact over two periods, “peace” and “conflict.” During peacetime, firms in each country produce goods and invest in capital, households consume, and countries trade internationally. Firms can invest in many types of capital—such as semi-conductor factories or oil reserves—and capital is sticky, so that investment during peacetime determines capital stocks during both peacetime and conflict. At the beginning of the conflict period, Foreign privately observes a geopolitical preference shock that determines its (purely non-economic) desire to, for example, invade another country. Home then makes a “trade threat” that specifies Home’s trade taxes as function of Foreign’s geopolitical action. Foreign chooses a geopolitical action, balancing its geopolitical desires with the economic incentives posed by Home’s threat. Finally, production, consumption, and trade—at the newly determined trade taxes—occur in the conflict period.
Within this setting, we study the design of two policy instruments that Home can use during peacetime to prepare for trade-mediated conflict: capital subsidies and trade taxes. Governments use both of these instruments in practice to strengthen their geopolitical bargaining position. The United States’s recent CHIPS and Science Act is a prime example. Domestically, it authorized tens of billions of dollars in subsidies for new microchip manufacturing plants in the US. Internationally, it banned US firms from producing certain chips in China and Russia, and the Biden administration subsequently restricted exports of some high-end AI chips to China. Many policymakers offered geopolitical rationales for these policies. For example, several Congresspeople argued that the US had “become dependent on foreign suppliers for semi-conductor chips – a position that severely weakens [its] standing on the world stage and exposes [it] to a significant vulnerability” (Katko et al., 2023).

Our first main result shows Home’s optimal capital subsidies are zero. This is counterintuitive since the private firms who invest in capital do not internalize how their investments impact Foreign’s geopolitical actions—either by changing the quantities traded given Home trade taxes or by influencing the taxes Home threatens. This suggests their investments have geopolitical externalities and so should be taxed or subsidized. In fact, such capital accumulation policies are sub-optimal. This is because Home’s trade threat already balances the economic cost of trade distortions against the geopolitical benefit of influencing Foreign’s actions. As a result, whatever marginal changes in Home’s trade under conflict are induced by additional Home investments have economic costs that, to first-order, offset their geopolitical benefits.

If capital subsidies are an inefficient way to prepare for geopolitical conflict, why are they used in practice? Our next set of results provides two possible answers by relaxing the model’s strong assumption that Home can credibly threaten any geopolitics-conditional trade taxes. First, we consider the possibility that Home cannot credibly threaten trade taxes that would lower its own economic welfare too much. For example, the EU decided not to impose any short-term sanctions on imports of natural gas from Russia following its invasion of Ukraine because member countries, particularly Germany, feared severe economic fallout (Moll et al., 2023). We show that in the presence of such a credibility constraint, capital accumulation policy during peacetime emerges as a second-best instrument. The optimal capital subsidies take a simple form; they set firms’ expectation of post-tax capital rental rates during conflict to what would be firms’ expectations of pre-tax rental rates if they placed counterfactually high probability on geopolitical actions for which Home’s credibility constraint binds. This policy therefore prepares Home for conflict by loosening its credibility

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1 We also characterize Home’s optimal trade threats. However, these are familiar from the literature on optimal sanctions, especially Becko (2024), and so are not our primary contribution.
constraint. In a stylized example, we show that these capital subsidies promote productive capacity in goods Home imports and discourage productive capacity in goods Home exports. In other words, capital accumulation policy leans away from comparative advantage.

A second consideration not present in our baseline model is that countries’ trade policies may be constrained by norms of the world trading system, such as non-discrimination. While the WTO’s national security exception allows for sanctions in response to certain foreign geopolitical actions, countries may be constrained in their ability to reward trading partners who abstain from unwanted geopolitical actions. Motivated by these concerns, we consider a second constraint on trade threats under conflict: Home can only set its tariffs freely if Foreign takes sufficiently undesirable geopolitical actions. This constraint again motivates Home capital subsidies during peacetime. The optimal subsidies promote investments that increase Home’s net exports of goods for which it anticipates large “missing trade taxes” under cooperative Foreign geopolitical behavior. This compensates for Home’s inability to reward Foreign directly through trade. In a stylized example, we show that these capital subsidies promote productive capacity in goods Home exports and discourage productive capacity in goods Home imports. In other words, capital accumulation policy leans into comparative advantage—the opposite of the credibility-constraint case.

The second part of the paper turns to peacetime trade policies. In studying the relationship between economic integration and geopolitical conflict, we connect to the longstanding theory that trade promotes peace.

“The natural effect of commerce is to lead to peace. Two nations that trade with each other become reciprocally dependent.” —Montesquieu (1748)

Our analysis of Home’s optimal peacetime trade policies makes this idea precise, formalizing one channel through which trade builds dependence and showing what sorts of dependence serve the geopolitical interests of a country considering economic integration.

The main contribution of this part of the paper is to characterize Home’s geopolitical motive for peacetime trade taxes. We show that trade interventions can serve a geopolitical purpose to the extent that they influence Foreign’s capital investments. Specifically, they target changes in investment that make world prices more sensitive to trade during conflict. Intuitively, by making world prices more sensitive to trade, Home can enhance the impact of its trade sanctions during conflict.

Surprisingly, the changes in Foreign capital that Home targets with its peacetime trade policy are not simply those that increase Foreign’s gains from trade during conflict—i.e. its economic incentive to comply with Home’s threats. On the one hand, Home targets changes

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2The English translation comes from Montesquieu et al. (1989).
in capital that raise Foreign’s gains from trade by making Foreign’s net export demand more inelastic. On the other hand, Home completely ignores changes in capital that raise Foreign’s gains from trade by influencing the quantities it trades under different sanction regimes. The rationale for this stark result is twofold. First, to the extent movements in Foreign capital influence traded quantities in ways that Home could alternatively influence using trade taxes under conflict, it prefers the latter, more direct instrument. Second, any other influences on traded quantities reflect the terms-of-trade effects of Foreign capital movements. These effects are already captured by the conventional dynamic terms-of-trade motive for peacetime trade policy, which Home ignores if it has no desire to redistribute economic welfare between itself and Foreign.³

Taken together, our results provide a coherent basis for peacetime geopolitical policies in countries that may impose trade sanctions in the future. Absent other market failures, countries need not use capital accumulation policies to strengthen their geopolitical bargaining position. However, players facing credibility constraints—perhaps such as the EU—can consider capital accumulation policies that lean against comparative advantage, particularly in core sectors. Countries—perhaps such as the US—whose trade threats are credible but who seek to abide by the WTO’s non-discrimination norm, can instead consider capital accumulation policies that lean into comparative advantage. Countries can also use peacetime trade policies to promote international dependence. These policies are effective to the extent that economic integration encourages foreign investments that makes world prices more sensitive to trade. However, if integration leads to investments that increase trade without making prices more sensitive—such as gas pipelines that uniformly reduce trade costs—then it may not serve a geopolitical purpose.

Related Literature

In the wake of global supply disruptions triggered by COVID-19 and Western sanctions on Russia, a growing literature has emphasized countries’ exposure to global supply chain risk and considered policies such as onshoring or friendshoring (Huang, 2017; Elliott et al., 2022; Baldwin et al., 2023). Relevant to our work, recent theoretical contributions by Traiberman and Rotemberg (2023), Grossman et al. (2023), and Grossman et al. (2023) show how various market failures—such as learning-by-doing and imperfect competition—can lead to excessive economic integration and rationalize re-shoring policies. While we consider similar policy

³To keep our analysis focused on trade rather than financial sanctions, we assume Home and Foreign are in financial autarky. This implies that dynamic terms-of-trade effects can in some cases also improve efficiency by compensating for missing financial markets. We extensively discuss this modeling decision in Section 2.2 and its consequences in Section 4.2.
instruments, we intentionally abstract from these frictions in order to isolate the explicitly geopolitical rationale for policy. One of this paper’s main contributions is provide a modeling framework that—by incorporating a geopolitical game into a model of international trade—allows us to study this rationale in a disciplined, micro-founded way.

Our emphasis on economic interdependence and geopolitical leverage harkens back to Hirschman (1945, 1958)’s analysis international trade as an instrument of national power. Within the economics literature, these themes have recently been picked up by Clayton et al. (2023). These authors study how a global hegemon can mitigate contracting frictions with, extract rents from, and influence the behavior of international firms by coordinating threats among domestic and domestically supplied firms. Although related in spirit, our framework significant differs from that of Clayton et al. (2023) in how it models both economic and geopolitical interactions. On the economic side, we consider a standard neoclassical model of production, investment, and trade and study conventional policy instruments; they adopt a game-theoretic framework that emphasizes contractability and coordinated threats in supply chains. On the geopolitical side, we microfound the desire to influence foreign economic activity by explicitly modeling a geopolitical game that economic considerations can influence; they model geopolitical preferences in reduced form, as pure externalities from economic activity in certain foreign sectors. These modeling differences lead us to substantially different and complementary results.

In studying international conflicts mediated through trade, we connect to the active literature on consequences and design of economic sanctions (Morgan et al., 2023; Bachmann et al., 2022; de Souza et al., 2023). Most directly, we build on Becko (2024)’s theory of optimal sanctions to characterize Home’s optimal sanctions during conflict (also see Osgood (1957) for an earlier related contribution). Our essential departure from this literature is that—rather than emphasizing policies during times of conflict—we ask how countries can prepare for or prevent conflict using forward-looking policies in peacetime.

Methodologically, our work builds on the seminal “production efficiency” result of Diamond and Mirrlees (1971) and related ideas in Atkinson and Stiglitz (1976), Cremer et al. (1998), and, more recently, Costinot and Werning (2018) and Donald et al. (2023). The unifying theme of these papers is to show that, in the presence of optimal second-best policies in one part of the economy, policies in another part of the economy can be set in a first-best way. At a high level, our result on laissez-faire capital subsidies under optimal trade threats follows a similar logic. We also move beyond this benchmark by augmenting the envelope-
theorem arguments recently used in this literature to study the case where Home’s trade threats are constrained, in which case Home’s optimal capital subsidies are non-zero.

**Paper outline.** Section 2 presents the model. Section 3 studies Home’s optimal capital accumulation policies, while Section 4 studies Home’s optimal trade policies. Section 5 concludes and describes next steps.

## 2 Model

We now present a model of trade and geopolitics with two economic periods: a period of peace and a period of geopolitical conflict. Private firms in each country make durable capital investments during peacetime that affect their country’s readiness to either impose or face trade sanctions during conflict. We study how a sanction-imposing government can use capital subsidies and trade taxes during peacetime to influence these investments and thereby shape the economic cost and geopolitical efficacy of their sanctions.

### 2.1 Environment and timing

There are three times $T = 0, 1, 2$, two countries $i$—Home (“$H$”) and Foreign (“$F$”)—and finitely many goods $g \in \mathcal{G}$ and types of capital $d \in \mathcal{D}$. Each country contains a representative household, goods producer, and capital producer.

At time 0, Home’s government announces three policies:

1. Ad-valorem subsidies $\{s_{H,d}^1\}_{d \in \mathcal{D}}$ on Home capital rentals at time 1.
2. Ad-valorem trade taxes $\{t_{H,g}^1\}_{g \in \mathcal{G}}$ levied at time 1.
3. A threat $\tau_H^2$ that commits Home to time-2 ad-valorem trade taxes $\{t_{H,g}^2\}_{g \in \mathcal{G}}$ as a function of a geopolitical action $a_F \in \mathcal{A}_F$ Foreign takes at the beginning of time 2.

Foreign engages in laissez-faire capital-formation and trade policies in all periods.\(^5\)

Time 1 represents a time of peace. At time 1, goods production $y_i^1$ and consumption $c_i^1$ occur in each country. Capital producers transform a vector of goods $\iota_i^1$ into a vector of capital $k_i^1$, which they own and rent to goods producers at prices $r_i^1$. In Home, rental prices are augmented by the capital subsidy $s_i^1$. Countries trade net exports $x_i^1$ subject to

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\(^5\)In the main text, we abstract from Foreign capital subsidies, Foreign trade taxes, and Home geopolitical actions. However, all of our results generalize to a richer setting—which will be covered in the appendices of future drafts—in which (a) Foreign simultaneously announces R&D subsidies and time-1 trade taxes at time 0, (b) Home’s threat commits it to a geopolitical action in addition to time-2 trade taxes, and (c) Home’s threat conditions not only on Foreign’s geopolitical action, but also its time-2 trade taxes, which are chosen at the same time.
Home’s trade taxes $t_i^1$. There is no intertemporal trade (see discussion below). All profits and government revenues are distributed lump-sum domestically.

Time 2 represents a time of conflict. At the beginning of time 2, Foreign observes an exogenous geopolitical preference shock $\theta_F \in \Theta_F$. Foreign then chooses a geopolitical action $a_F$ from a finite set $A_F$. This determines Home’s time-2 trade taxes $t_H^2 = \tau_H^2(a_F)$. Production, consumption, and trade then occur as at time 1. Capital does not depreciate and there is no further investment, so $k^2_i = k^1_i$.

![Figure 1: Timing of model](image)

### 2.2 Discussion

Before stating the equilibrium conditions, we highlight a few key modeling choices.

**Asymmetry of Home and Foreign.** In the model, Home has a first-mover advantage, making threats $\tau_H^2$ that Foreign takes as given. We focus our analysis on this case for two reasons. First, it is relevant, since many real-world geopolitical negotiations feature large, repeat players—such as the United States—who can plausibly commit to trade threats. Second, it allows us to isolate novel economic mechanisms that would exist alongside other motives in more complex models of international bargaining.

**Punishments in equilibrium.** Because Home does not observe Foreign’s geopolitical preference shock, it will sometimes end up having to “make good” on severe trade threats that would lie off of the equilibrium path in models with perfect information. We view this as a realistic feature of the model. For example, the European Union and the United States imposed harsh trade sanctions on Russia in response to its invasion of Ukraine, but these sanctions appear to have had little effect on Russia’s war effort.

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6The other differences between countries are inessential and can be relaxed to allow for Foreign capital subsidies, Foreign trade taxes, and Home geopolitical actions.
Choice of instruments. It may appear surprising that we allow the Home planner to use capital subsidies but not more conventional instruments such as consumption or production taxes. In many environments, consumption and production taxes dominate policies—such as capital subsidies—that place wedges between firms (Diamond and Mirrlees, 1971). Our choice of instruments reflects that, by the first welfare theorem, economic activity in each countries is Pareto efficient conditional on trade and capital. Home would therefore not use consumption or production taxes if they were available. We find that Home does offer non-zero capital subsidies, violating production efficiency to strengthen its geopolitical position.

Financial autarky. We assume that countries are in financial autarky, neither saving and borrowing nor trading insurance against Foreign’s preference shock. We thereby set aside a multitude of considerations related to international finance. For example, Home may want to borrow from Foreign at time 1 so that it can condition repayment on Foreign’s geopolitical actions by freezing Foreign assets. Alternatively, if debts must be repaid, Home may prefer to indebt Foreign to raise its time-2 marginal consumption utility and hence its susceptibility to trade sanctions. These issues are important and complex, but beyond our scope.

Non-tradability of capital. The essential distinction between goods and capital in our model is that capital is durable. However, we also assume that capital is non-tradable. This assumption is natural for types of capital like microchip factories, but not for other types, such as oil stockpiles. Importantly, our results do not rely on the non-tradability of capital, since we can nest trade in (any type of) capital by introducing another good, “traded capital.” Traded capital is produced by goods producers in the exporting country using the same inputs as capital producers use to produce regular capital, and capital producers in the importing country can convert traded capital to regular capital one-for-one.

Separability of economics and geopolitics. We assume in the main text that economics and geopolitics interact only through Home’s trade threat. In particular, we abstract from any production technologies or spending demands that depend directly on geopolitical actions. This prevents us from speaking to issues such as trade in arms and military attacks on economic bottlenecks. Future drafts will extend our results to the more general case.

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7 Bianchi and Sosa-Padilla (2024) study the optimal use of asset freezes as sanctions, although they do not model foreign geopolitical actions or Home’s ex-ante incentives to promote foreign asset accumulation.
8 We additionally assume that capital is taxed domestically whereas goods are not, but this is without loss of generality as discussed above.
2.3 Equilibrium conditions

We begin by specifying the equilibrium conditions given Home’s capital subsidies \(s_{1H}\), first-period trade taxes \(t_{1H}\), and second-period trade tax threat \(\tau_{2H}\). The following section turns to Home’s optimal policy problem.

Below, we index time-2 variables by the state of the world \(\omega \in \Omega\) to express their dependence on Foreign’s preference shock. For technical convenience, we assume \(\Omega\) is finite.

Households. Households choose net consumption in the first period and in each state of the world in the second period, subject to a budget constraint that reflects their ownership of domestic profits and government revenues. We assume the existence of domestically-traded intertemporal and state-contingent bonds, so that households face an expected lifetime budget constraint. Since countries are in financial autarky and bonds are in net zero supply, bonds are not traded in equilibrium.

\[
\begin{align*}
&c_{1}, \{c_{2}(\omega)\}_{\omega \in \Omega} \in \arg \max_{c_{1}, \{c_{2}(\omega)\}_{\omega \in \Omega}} u_{i}^{1}(c_{1}) + \beta \mathbb{E}[u_{i}^{2}(c_{2}(\omega))]
\text{s.t. } p_{1}^{i} \cdot c_{1} + \mathbb{E}[p_{2}^{2}(\omega) \cdot c_{2}(\omega)] \leq \pi_{i} + R_{i}
\pi_{i} = p_{1}^{i} \cdot (y_{1}^{i} - u_{1}^{i}) + s_{1}^{i} r_{1}^{i} \cdot k_{1}^{i} + \mathbb{E}[p_{2}^{2}(\omega) \cdot y_{2}^{i}(\omega)]
R_{i} = \frac{p_{1}^{i} t_{1}^{i}}{1 + t_{1}^{i}} \cdot x_{1}^{i} - s_{1}^{i} r_{1}^{i} \cdot k_{1}^{i} + \mathbb{E} \left[ \frac{p_{2}^{2}(\omega) t_{2}^{i}(\omega)}{1 + t_{2}^{i}(\omega)} \cdot x_{2}^{i}(\omega) \right]
\end{align*}
\]

Above, \(u_{i}^{T}\) is the utility function of the household in \(i\) at time \(T\) and \(\beta > 0\) is its discount rate. Recall that \(s_{1}^{i} t_{1}^{i} = t_{2}^{i}(\omega) = 0\).

Firms. Goods producers rent capital and produce net output at each time and state of the world, subject to a production possibilities frontier \(G_{1}^{i}(y_{i}, k_{i}^{D,1}) \leq 0\). We let \(k_{i}^{D,1}\) and \(k_{i}^{D,2}(\omega)\) denote goods producers’ capital demands.

\[
\begin{align*}
y_{1}^{i}, k_{i}^{D,1} & \in \arg \max_{y, k} p_{1}^{i} \cdot y - r_{1}^{i} \cdot k \quad \text{s.t. } G_{1}^{i}(y, k) \leq 0
y_{2}^{i}(\omega), k_{i}^{D,2}(\omega) & \in \arg \max_{y, k} p_{2}^{2}(\omega) \cdot y - r_{2}^{i}(\omega) \cdot k \quad \text{s.t. } G_{2}^{i}(y, k) \leq 0
\end{align*}
\]

Forward-looking capital producers transform inputs into capital according to a capital production possibilities frontier \(\Lambda_{1}^{1}(k_{i}, \iota_{i}) \leq 0\). They own and rent this capital to domestic goods producers in both periods. Investment occurs only in the first period and capital does
not depreciate.

\[ \arg \max_{t^1, k^1, \{k^2(\omega)\}_{\omega \in \Omega}} \quad r^1(1 + s^1_t) \cdot k^1 - p^1_t \cdot l^1 + \mathbb{E}[r^2_t(\omega) \cdot k^2(\omega)] \]

\[ \operatorname{s.t.} \quad \Lambda^1_t(k^1, l^1) \leq 0 \]
\[ \forall \omega \in \Omega, \quad k^2(\omega) = k^1 \]  

**(3)**

**Trade balance and market clearing.**  Home and Foreign prices are consistent with the same world prices, given Home’s trade taxes.

\[ p^1_H = p^1_F(1 + t^1_H) \quad \text{and} \quad p^2_H(\omega) = p^2_F(\omega)(1 + t^2_H(\omega)) \]  

**(4)**

Trade balance holds at world prices at every time and every state of the world.\(^9\)

\[ p^1_F \cdot x^1_F = 0 \quad \text{and} \quad p^2_F(\omega) \cdot x^2_F(\omega) = 0 \]  

**(5)**

Domestic capital markets and domestic and international goods markets clear.

\[ k^1_i = k^D_i \quad \text{and} \quad k^2_i(\omega) = k^D_i(\omega) \]

\[ y^1_i = c^1_i + l^1_i + x^1_i \quad \text{and} \quad y^2_i(\omega) = c^2_i(\omega) + x^2_i(\omega) \]  

**(6)**

**Geopolitics.**  After observing its geopolitical preference shock \( \theta_F \), Foreign chooses a geopolitical action \( a_F \) to maximize the sum of its household’s time-2 consumption utility and an additively-separable geopolitical utility \( v_F(a_F, \theta_F) \). Its geopolitical action affects consumption utility through Home’s trade tax response, which is given by the threat function \( \tau^2_H \) applied to \( a_F \). This threat must lie within a set of feasible threats \( T^2_H(k^1_i)_{i = H,F} \) that in general can depend on capital in both countries. We assume that trade taxes, in conjunction with Home and Foreign capital formed at time 1, uniquely determine the time-2 equilibrium (up to price indeterminacy), with consumption and exports in country \( j \) and state \( \omega \) given

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\(^9\)In static trade models, trade balance is a result implied by household budget constraints. In our setup, trade balance is instead an assumption that reflects financial autarky (across time and states of the world).
by the functions $C^2_j(t^2_H, \{ k^2_i(\omega) \}_{i=H,F})$ and $X^2_j(t^2_H, \{ k^2_i(\omega) \}_{i=H,F})$.

$$a_F(\omega) \in \arg \max_{a_F \in A_F} u^2_F \left( C^2_H(t^2_H, \{ k^2_i(\omega) \}_{i=H,F}) \right) + v_F(a_F, \theta_F(\omega))$$

s.t. \[ t^2_H = \tau^2_H(a_F) \]

$$t^2_H(\omega) = \tau^2_H(a_F(\omega))$$

$$\tau^2_H \in T^2_H(\{ k^1_i \}_{i=H,F})$$

(7)

**Definition 1.** An equilibrium is a profile \( \{ s^1_H, t^1_H, t^2_H(\omega), \tau^2_H, p^1_i, p^2_i(\omega), r^1_i, r^2_i(\omega), c^1_i, c^2_i(\omega), y^1_i, y^2_i(\omega), k^{D,1}_i, k^{D,2}_i(\omega), \nu^1_i, k^1_i, k^2_i(\omega), x^1_i, x^2_i(\omega), a_F(\omega) \}_{i=H,F; \omega \in \Omega} \) that satisfies Equations 1–7 for all \( i = H, F \) and \( \omega \in \Omega \).

### 2.4 Home’s policy problem

We now turn to the Home’s policy problem at time 0. As in Foreign, Home welfare is equal to the sum of its household’s consumption utility and an additively-separable geopolitical utility \( v_H(a_F) \). We additionally allow for the possibility that Home places non-zero weights \( \lambda^E \) and \( \lambda^G \) on Foreign economic and geopolitical utility, respectively. This is important because it implies our results characterize the behavior of not only selfish hegemons but also altruistic “global policemen.”

In order to maximize this objective, Home chooses time-1 capital subsidies \( s^1_H \) and trade taxes \( t^1_H \) and commits to a trade threat \( \tau^2_H \) that lays out trade taxes \( t^2_H \) as a function of Foreign’s geopolitical action. While we assume Home’s first-period policies are unconstrained, recall we require that Home’s trade threat falls within a feasible set \( T^2_H(\{ k^1_i \}) \). This set can, in general, depend on both countries’ capital in the resulting equilibrium, allowing us to encode policy constraints that vary with the state of the global economy. Since capital is itself determined by these policies, the feasible-threat function and the equilibrium conditions together define a set of implementable policies \( P \). Given its policy choices, we allow Home to select any profile of consumption and Foreign geopolitical actions that—conditional on policy—is consistent with the equilibrium conditions. We denote the set of such profiles by \( \mathcal{E}(s^1_H, t^1_H, \tau^2_H) \).

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10 We provide sufficient conditions for uniqueness in Appendix A.1.
Definition 2. An equilibrium is Home-optimal if it satisfies Equation 8.

Throughout the paper, and without further statement, we study Home-optimal equilibria. Our results rely on standard neoclassical regularity conditions stated in Appendix A.1. For simplicity, we additionally assume Foreign’s geopolitical preference shock has a wide enough support that, in any Home-optimal equilibrium, all Foreign actions occur with positive probability.

2.5 Example: Quasi-linear preferences with fixed production

Having laid out the general model, we now introduce a parametric special case that we use to build intuition throughout the paper. While stylized, this example is rich enough to speak to many topical issues including the role of “core” goods (those demanded inelastically), strategic stockpiles, and trade in upstream inputs.

On the household side, we suppose utility is quasi-linear. That is, for all times $T$ and countries $i$,

$$u_i^T(c) = c_0 + \sum_{g \in G \setminus \{0\}}^{} u_{ig}^T(c_g)$$

for some increasing, concave, and smooth sub-utility functions $u_{ig}^T$. We further focus on the limit where conflict is short-lived (i.e. $\beta \to 0$).

On the production side, we suppose goods are organized into simple supply chains. Nonzero even-numbered goods $g$ are “upstream goods” of which each country $i$ has an exogenous endowment $\bar{y}_{ig}^T$ at each time $T$. Upstream goods $g$ can be consumed directly at any time or converted one-for-one into capital of a type $d_{g-1}$ at time 1. Odd-numbered goods $g$ are “downstream goods” produced using capital of type $d_g$ according to an increasing, concave production function $f_{ig}$ in each country $i$. That is,

$$y_{ig}^T = \bar{y}_{ig}^T \quad \text{and} \quad k_{id_{g-1}}^T = l_{ig}^1 \quad \text{for } g \text{ even}, \quad \text{and} \quad y_{ig}^T = f_{ig}(k_{id_g}^T) \quad \text{for } g \text{ odd.} \quad (11)$$

¹¹Formally, the set of capital types is $\mathcal{D} = \{d_{g-1} \mid g \in G, \ g \text{ non-zero and even}\}$ and the economy has goods
On the geopolitical side, we assume Foreign only has two actions: aggression or restraint. Home experiences a higher geopolitical utility when Foreign chooses restraint than when it chooses aggression, i.e. $v_H(\text{restraint}) > v_H(\text{aggression})$.

Throughout, we assume that the second derivative of Home’s objective function in the second period is negative.\footnote{Formally, we assume that the second derivative of $u^2_{HG} (y_{HG}^2 + x_F^2) + \rho u^2_{FG} (y_{FG}^2 - x_F^2) - (1 - \rho) \frac{\partial u^2_{FG}}{\partial x} y_{FG}^2$ with respect to $x_F$ is negative for all $\rho \in [\rho(a_A), \rho(a_R)]$ where $\rho(m)$ is the endogenous weight Home puts on period 2 economic welfare in Foreign defined in Appendix B and $y^2_{FG}$ is the equilibrium level of production of country $i$ in good $g$.}

## 3 Optimal capital subsidies

We now turn to the analysis of Home’s optimal capital subsidies. We characterize these subsidies under the assumption that Home also engages in optimal trade policy, which we study in Section 4. Our results highlight the dependence of capital subsidies on Home’s ability to commit to credible time-2 trade threats.

### 3.1 Capital subsidies with unrestricted trade threats

We begin with the case in which Home’s trade threats are unrestricted.

**Assumption 1.** Home can commit to credibly threaten any ad-valorem trade taxes in response to any Foreign geopolitical action, i.e.

$$T^2_H([{k^1_i}]) = (\mathbb{R}_{\geq 1}^G)^{AF}$$

(12)

It is natural to suspect that—even with unconstrained trade threats—Home should use capital subsidies to realign the incentives of atomistic capital producers. Indeed, these capital producers fail to internalize the effect their investments have on the probability of conflict by shifting Foreign economic welfare under any given Home trade taxes. Counter to this informal argument, our first result shows that Home engages in laissez faire domestic policies when its trade threats are unconstrained.

**Proposition 1.** Under Assumption 1, Home capital subsidies are zero.
To understand Proposition 1, it is helpful to revisit the informal argument in favor of capital subsidies presented before the proposition. We begin by breaking the Home planner’s problem into two steps: First, choosing a level of Home capital $k^1_H$ to implement and, second, choosing a trade-and-geopolitics profile $\{x^1_F, x^2_F(\omega), a_F(\omega)\}_{\omega \in \Omega}$ among those implementable given Home capital $k^1_H$—call them $\tilde{E}(k^1_H)$—to maximize Home’s objective $W_H$.\(^{13}\)

$$\max_{k^1_H} \max_{\{x^1_F, x^2_F(\omega), a_F(\omega)\} \in \tilde{E}(k^1_H)} W_H(\{x^1_F, x^2_F(\omega), a_F(\omega)\}, k^1_H) \quad (13)$$

Let $\{x^1_F(k^1_H), x^2_F(k^1_H; \omega), a_F(k^1_H; \omega)\}_{\omega \in \Omega}$ denote the inner problem’s solution given $k^1_H$.

The informal argument conceptualizes the welfare impacts of a change in Home capital as the sum of (a) the direct effect of $k^1_H$ on $W_H$ and (b) the indirect effect of $k^1_H$ on $W_H$ through the inner solution $\{x^1_F(k^1_H), x^2_F(k^1_H; \omega), a_F(k^1_H; \omega)\}_{\omega \in \Omega}$. One may show that (a) corresponds to the private incentives of goods producers, whereas (b) is uninternalized. Capital subsidies are therefore justified precisely to the extent that welfare responds to the capital-induced change in trade and geopolitics.

In contrast to the informal argument, the proof of Proposition 1 draws on the envelope theorem to show that the indirect welfare effects of a change in Home capital are precisely zero when Home trade taxes are unconstrained. Concretely, the envelope theorem implies that Home capital’s indirect welfare effects are proportional to their effect on the implementable set $\tilde{E}(k^1_H)$. The key step of the argument is to recognize that, with unconstrained trade taxes, $\tilde{E}(k^1_H)$ in fact does not depend on $k^1_H$. This is because (i) Foreign’s geopolitical action is determined by Home’s threatened trade vector under each action, and (ii) the set of implementable trade vectors is simply Foreign’s offer curve, which does not depend on Home technology. Since $\tilde{E}(k^1_H)$ is invariant to $k^1_H$, Home capital’s effect on welfare is fully captured by its direct effect on $W_H$, which firms internalize.

Another way to understand Proposition 1 is as a manifestation of the targeting principle. This interpretation notes that the only mode of economic interaction between Home and Foreign is international trade. Moreover, for any level of trade that Home can implement by subsidizing Home capital investments, Home can implement the same level of trade directly through Foreign-action-contingent trade policies. This direct approach is more efficient as it does not distort Home production decisions given trade.

\(^{13}\)Equation 13 is a complete description of Home’s problem because the first welfare theorem implies that, given capital, trade, and geopolitical actions, markets implement efficient allocations in each country.
3.2 Capital subsidies with limited credibility

We have just provided an assumption under which capital subsidies are an inefficient way to affect geopolitical outcomes. And yet, policymakers often cite geopolitical considerations to justify capital accumulation policies, such as national petroleum reserves or semiconductor R&D subsidies.

With an eye toward rationalizing these real-world policy decisions, we now depart from the assumption of unconstrained trade policy. We show that, away from this benchmark, capital subsidies operate in a second-best capacity that helps to compensate for constrained trade policies. Our results highlight how the optimal capital subsidies depend on the nature of these constraints.

We begin by considering the case where Home’s trade threats are not fully credible. Concretely, we suppose that after Foreign selects a geopolitical action, Home politicians will “conveniently misinterpret” this action if Home’s welfare loss under its threatened trade policies are too large relative to its welfare under its threatened trade policies at a different action. Formally, we model this constraint as a restriction on Home’s feasible trade threats.

Assumption 2. Home can threaten any Foreign-geopolitical-action conditional-trade taxes for which the gap between Home’s time-2 economic welfare across geopolitical actions is sufficiently small.\(^\text{14}\) I.e. for some \(\delta > 0\),

\[
\mathcal{T}_H^2(\{k^i\}) = \left\{ \tau^2_H : \mathcal{A}_H \to \mathbb{R}_{\geq -1} \mid \max_{a_F \in A_F} u^2_H(C^2_H(\tau^2_H(a_F), \{k^1_i\})) - \min_{a_F \in A_F} u^2_H(C^2_H(\tau^2_H(a_F), \{k^1_i\})) \leq \delta \right\}
\]

Under Assumption 2, Proposition 1 no longer applies and, in general, optimal capital subsidies can be non-zero. This is because when Home’s credibility constraint binds, its trade threats can no longer implement all maps from Foreign geopolitical actions to trade outcomes on Foreign’s offer curve. Moreover, which Home threats are credible depends on Home capital—or, in the terminology of Equation 13, \(\bar{\mathcal{E}}(k^1_H)\) depends on \(k^1_H\)—and private firms do not internalize this impact.

Our next result formalizes the precise direction in which Home’s planner seeks to shift capital supply. For simplicity, we state the result in the case where there are unique best and worst actions from the perspective of Home economic welfare. For the general case, see Appendix A.4.

\(^{14}\)An equally plausible assumption is that Home cannot threaten any trade taxes under which its economic welfare would be sufficiently lower than under its economic-welfare-maximizing trade taxes (whether or not these taxes are part of Home’s threat). This assumption generates qualitatively similar results except that the “alternative stochastic discount factor” in Proposition 2 will typically be negative for some states.
Assumption 3. There are unique best and worst actions for Home economic welfare, i.e. \( \pi_F \) and \( a_F \) such that for all other \( a_F \in \mathcal{A}_F \),
\[
u^2_H(C^2_H(\tau^2_H(a_F), \{k^1_i\})) < \nu^2_H(C^2_H(\tau^2_H(a_F), \{k^1_i\})) < \nu^2_H(C^2_H(\tau^2_H(\bar{a}_F), \{k^1_i\}))
\]

Proposition 2. Under Assumptions 2 and 3, post-subsidy time-1 capital prices in Home are equal to pre-subsidy time-1 capital prices evaluated under an alternative measure over time-2 states \( \bar{\mu} \). This alternative measure is equal to the true probability measure \( \mu \) except for that it assigns less mass to states in which the best action \( \pi_F \) occurs and more mass to states in which the worst action \( a_F \) occurs.\(^{15}\) Formally,
\[
r^1_H s^1_H = \mathbb{E}_{\bar{\mu}}[r^2_H(\omega)] - \mathbb{E}[r^2_H(\omega)]
\]

Proposition 2 says that Home should use capital subsidies to shift capital producers’ focus away from high-Home-welfare states toward low-Home-welfare states. Doing so results in a capital stock that reduces Home’s time-2 economic welfare gap between those states, loosening the credibility constraint. Since differences in Home’s economic welfare across states of the world result from its time-2 trade policies, this policy promotes a capital stock that substitutes for goods made scarce by trade sanctions.

One interesting aspect of Proposition 2 is that it implies Home could also implement the optimal level of capital with an information treatment. If Home firms simply assigned higher likelihood to the states of the world with the lowest Home welfare and lower likelihood to the states of the world with the highest Home welfare, then they would act as under the optimal policy. This offers a new rationale for geopolitical “fear-mongering” by policymakers.

In order to spell out Proposition 2’s concrete implications for capital subsidies, we now turn to the concrete special case introduced in Section 2.5.

Example. Suppose Home sanctions for aggression are severe enough that Home has lower economic utility when Foreign chooses aggression than when it chooses restraint. Then for any \( g \in \mathcal{G}\backslash\{0\} \), the direction of trade in \( g \) is consistent across aggression and restraint, and Home’s capital accumulation subsidies can then be characterized by the direction of trade of downstream (i.e. odd-numbered) goods. Specifically,

1. If Home imports good \( g \), then Home subsidizes productive capacity of good \( g \) domestically, i.e. \( s^1_{dg} > 0 \).

\(^{15}\)In general, the weights \( \bar{\mu}(\omega) \) can be negative. They are positive if the geopolitical action that gives Home the highest economic utility is likely enough.
2. If Home exports good $g$, then Home taxes productive capacity in good $g$ domestically, i.e. $s_{d_g}^1 < 0$.

The logic behind capital subsidies in the example is that Home should promote its productive capacity for goods that are scarce under aggression and discourage production for goods that are abundant under aggression. This scarcity and abundance results from Home’s own time-2 trade taxes, which—as we show in Section 4—target goods for which Foreign prices are more manipulable, i.e. those Foreign net exports inelastically. With quasi-linear utility, Home buys less of the goods $g \in \mathcal{G}\setminus\{0\}$ it imports from Foreign. Increasing productive capacity in these goods and decreasing it in others narrows the gap in Home utility between aggression and restraint, loosening its credibility constraint. We show in Appendix B that the size of the subsidy for capital type $d_g$ is increasing in Home’s coefficient of relative risk aversion for $g$, since high risk aversion causes Home prices to spike when a good becomes scarce. Thus, capital subsidies should especially target productive capacity for goods for which Home utility is sensitive.

One recent example of credibility constraints may be the European Union’s response to Russia’s invasion of Ukraine. EU sanctions on natural gas imports from Russia were severely delayed by internal debates about the economic cost of such sanctions for member states, especially Germany. This suggests the EU could not have credibly threatened harsher sanctions. Our results therefore imply that—had the EU anticipated the possibility of Russia’s invasion—it should have used subsidies beforehand to promote its energy independence. Perhaps motivated by similar credibility constraints, the US maintains a strategic petroleum reserve in large salt caverns along the Gulf of Mexico. It has recently drawn from these reserves to compensate for rising oil prices caused by Western sanctions on Russia.

### 3.3 Capital subsidies with non-discrimination constraints

Another salient constraint on time-2 trade policies is international rules and norms such as non-discrimination. These rules do not always apply, since countries can invoke the WTO’s national security exception in response to certain foreign geopolitical actions, as the US and EU have recently done to justify sanctions on Russia. However, it is unclear whether countries could reward particular trading partners for “good geopolitical behavior” without facing resistance from others. Such rewards may also be politically infeasible within the rewarding country.

Motivated by these considerations, we consider a second constraint on trade policy. Specifically, we assume that Home’s time-2 trade taxes are constrained to free trade if For-
eign takes certain geopolitical actions. In line with the WTO’s national security exception, we assume that Home sets trade taxes freely only if Foreign takes a geopolitical action that gives Home sufficiently low geopolitical utility.

**Assumption 4.** Home can threaten any Foreign-geopolitical-action-conditional trade taxes for which Home trade taxes are zero when Foreign’s geopolitical action gives Home high enough geopolitical utility. Formally, there exists a cutoff \( \tau_H \) for which

\[
T_H^2(\{k^i\}) = \left\{ \tau_H^2 : \mathcal{A}_F \to \mathbb{R} \geq -1 \mid \forall a_F \in \mathcal{A}_F \text{ s.t. } v_H(a_F) \geq \tau_H, \tau_H^2(a_F) = 0 \right\}
\]

(17)

In the presence of this constraint on trade threats, Home capital subsidies again function in a second-best role. Our next result characterizes their optimal level. For simplicity, we state it in the \( \beta \to 0 \) limit where the conflict period is short. For the general case, see Appendix A.5.

**Proposition 3.** Under Assumption 4, Home technology subsidies satisfy

\[
s_{H,d}^1 s_{H,d}^1 = E \left[ \mathbb{1}_{v_F(a_F(\omega)) \geq \tau_F} t_H^{2*}(\omega)p_F(\omega) \frac{dx^{2,FT}_{H,d}}{dk_{H,d}^2} \right] + o(\beta^2)
\]

(18)

where \( t_H^{2*}(\omega) \) is Home’s unconstrained-optimal trade tax in state \( \omega \), given the actual values of policy-relevant elasticities, and where \( dx^{2,FT}_{H,d}/dk_{H,d}^2 \) is the marginal effect of type-\( d \) Home technology on Home net exports under free trade.

The idea behind this result is straightforward: While Home is unable to affect trade directly—i.e., using trade taxes—under some Foreign actions, it can still affect trade indirectly by subsidizing the formation of capital. Specifically, Home subsidizes the formation of capital types that, in general equilibrium, increase Home’s net exports of the goods whose net exports Home would have liked to promote directly, with trade taxes. The precise amount by which an additional unit of net exports increases Home welfare is revealed by Home’s unconstrained-optimal export subsidy (or import tariff), which explains the relative weight placed on exports of different goods in Equation (18).

We spell out Proposition 3’s concrete implications for capital subsidies in the context of our stylized example.

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16We could alternatively assume that Home’s trade taxes are constrained to a different, non-zero level, such as the MFN level. In this case, we would replace Home’s unconstrained-optimal trade tax \( t_H^{2*}(\omega) \) in Equation 18 with the gap between \( t_H^{2*}(\omega) \) and Home’s constrained tariff level; this gap would no longer simply equal \( t_H^{2*}(\omega) \) itself.

17We characterize Home’s unconstrained trade taxes in terms of policy-relevant elasticities in Section 4.
Example. Suppose that time-2 Home trade policy is constrained to free trade if and only if Foreign chooses restraint, and that Home’s unconstrained policy would reward Foreign for restraint more than does free trade. Then Home’s capital subsidies can be characterized by the direction of trade of downstream (i.e. odd-numbered) goods under free trade. Specifically,

- If Home imports good \( g \), then Home taxes productive capacity of good \( g \) domestically, i.e. \( s_{d_g}^1 < 0 \).
- If Home exports good \( g \), then Home subsidizes productive capacity of good \( g \) domestically, i.e. \( s_{d_g}^1 > 0 \).

These subsidies contrast sharply with those Home uses when its credibility is limited. When Home faces credibility constraints, it typically promotes investment in net imports—leaning away from comparative advantage—whereas when Home faces non-discrimination constraints, it typically promotes investment net exports—leaning into comparative advantage. Doing so worsens Home’s terms of trade with Foreign, allowing Home to “reward” Foreign restraint without using trade taxes.

This rationale for industrial policy echoes the historical experience of Haiti around the turn of the 19th century. Although Haiti was technically a colony of France, its military dictator Toussaint Louverture ruled with significant autonomy after leading a slave revolt that overthrew the former colonial government dominated by white plantation owners. The colonial system restricted Haitian trade policy, but Louverture used his domestic leeway to institute forced labor and enact other policies that promoted sugar exports to France. One interpretation is that these policies provided France with an economic incentive to abstain from invading, deposing Louverture, and reestablishing full slavery.\(^{18}\)

4 Optimal trade taxes

Our characterizations of optimal capital accumulation policies in Section 3 rely critically on the assumption that the Home planner also uses trade policy at times 1 and 2. In particular, the sub-optimality of capital subsidies—shown in Proposition 1—results from the Home planner’s ability to achieve the same goals by using trade taxes as a more direct instrument.

We now turn to the design of these trade taxes themselves. We focus our analysis on the case where Home’s trade threats are unconstrained (i.e. Assumption 1), since this scenario is already enough for Home to engage in non-trivial trade policies. While we characterize

\(^{18}\)See Girard (2011) for a discussion of Louverture’s actions leading up to Haitian independence.
Home trade taxes at both time 1 and time 2, we emphasize the former since the latter is similar to Becko (2024)’s analysis of optimal sanctions.

4.1 Trade taxes with exogenous Foreign capital

We begin by considering the case in which Foreign’s capital stock is exogenously fixed. This case is of limited direct interest as we show that Home’s time-1 trade policies are not affected by geopolitical considerations. However, it clarifies that Home’s geopolitical motive for peacetime trade policy hinges on its ability to influence Foreign capital—a topic we turn to in the following section.

**Assumption 5.** Foreign has a capital endowment and no ability to produce new capital. Formally, there exists \( \bar{k}_F \) such that

\[
\Lambda_F(k_F, \iota) \leq 0 \iff k_{F,d} \leq \bar{k}_{F,d} \quad \text{for all} \quad d \in D
\]  (19)

We now introduce Foreign’s static terms-of-trade elasticities, which we will show serve as sufficient statistics for Home’s optimal trade policies. These elasticities are in general endogenous equilibrium objects that must be evaluated under Home’s optimal policy.

As a preliminary step, note that since Foreign’s set of geopolitical actions, \( \mathcal{A}_F \), is finite, geopolitical actions can be enumerated as \( a_F(1), ..., a_F(M) \). For later use, we assume these actions are ordered according to Home’s geopolitical preference, i.e. \( v_H(a_F(m)) \leq v_H(a_F(m + 1)) \). With a slight abuse of notation, we let \( x_F^1(m) \equiv \mathcal{X}_F^1(\tau_H^2(a_F(m)), \{k^1_i\}_{i=H,F}) \) and \( \tau_H^2(m) \equiv \tau_H^2(a_F(m)) \) denote Foreign’s net exports and Home’s trade threat, respectively, under action \( a_F(m) \).

Turning now to Foreign’s terms-of-trade-elasticities, we denote by \( \Sigma_g^1 \) the effect of a change in Foreign net exports of \( g \) at time 1 on its terms of trade at time 1, and we similarly denote by \( \Sigma_g^2(m) \) the effect of a change in Foreign net exports of \( g \) at time 2 on its terms of trade at time 2, conditional on Foreign having taken the geopolitical action \( a_F(m) \).

\[
\Sigma_g^1 = \sum_{g' \in \mathcal{G}} \frac{x_{F,g'}^1(x_F^1, k_F^1)}{\partial x_{F,g}^1} \frac{\partial \tilde{p}_{F,g'}^1(x_F^1, k_F^1)}{\partial x_{F,g}^1} \quad \Sigma_g^2(m) = \sum_{g' \in \mathcal{G}} \frac{x_{F,g'}^2(m)}{\mu(m) \partial x_{F,g}^2(m)} \frac{\partial \tilde{p}_{F,g'}^2(x_F^2(m), k_F^1)}{\partial x_{F,g}^2(m)}
\]  (20)

Above, \( \mu(m) \) is the probability action \( a_F(m) \) occurs in equilibrium, and the functions \( \tilde{p}_F^T(x_F, k_F) \) are Foreign’s inverse net export supply curve, given capital \( k_F \). Because For-

19The inequality below implicitly assumes Foreign capital may be freely disposed.
eign’s economy is efficient conditional on trade and capital, its goods prices are proportional to marginal economic values of imports; we therefore define \( p_T^F(x_F, k_F) \) as the negative of the partial derivative with respect to net exports of its trade-and-capital-contingent economic utility \( V_T^F(x_F, k_F) \) (its “Meade utility”).\(^{20}\)

Our next result characterizes Home’s optimal trade policies in terms of these elasticities.

**Proposition 4.** Under Assumptions 1 and 5, there exist weights \( \lambda_F^1 \), \( \lambda_F^2(m) \), and \( \Delta \kappa(m) \) such that Home’s first-period trade taxes and second-period trade threats satisfy

\[
\begin{align*}
t_{H,g}^1 &= \lambda_F^1 (1 + \Sigma_g^1) \\
t_{H,g}^2(m) &= \lambda_F^2(m)(1 + \Sigma_g^2(m)) - \Delta \kappa(m) \Sigma_g^2(m)
\end{align*}
\]

up Lerner symmetry.\(^{21}\)

The proposition says that—when Foreign capital is fixed—Home trade policies simply manipulate terms of trade, but to a degree that in general depends on time and Foreign’s geopolitical action. The degree of terms-of-trade manipulation, captured by \( \lambda_F^1 \), \( \lambda_F^2(m) \), and \( \Delta \kappa(m) \) reflects three different considerations. First, Home manipulates terms of trade less to the extent it directly values Foreign economic welfare (i.e. \( \lambda^F \) is large). Second, Home manipulates terms of trade more in response to Foreign actions it wishes to disincentivize and less in response to Foreign actions it wishes to incentivize—an effect captured by \( \Delta \kappa(m) \). Third, even if Home has no geopolitical preferences and values Foreign consumption equally to its own, it may still manipulate terms of trade as a second-best way of compensating for missing financial markets. Intuitively, when \( \lambda_F^1 \neq \lambda_F^2(m) \) for some \( m \), Home can simulate trade across time or states of the world by worsening its terms of trade in one time or state and improving its terms of trade in another.

Notably, Proposition 4 shows that when Foreign capital is fixed, Home’s time-1 trade policy is not directly affected by the geopolitical game at time 2. Home simply manipulates its terms of trade with a weight that depends on how much it values Foreign economic welfare. This reflects a targeting principle: When Foreign capital is fixed, Home policy only affects

\(^{20}\)We implicitly assume a constant of proportionality of one between prices and marginal utility. This is without loss of generality since prices appear in both the numerator and the denominator of \( \Sigma_g^1 \) and \( \Sigma_g^2(m) \). The Meade utility functions of country \( i \) are defined by

\[
\begin{align*}
V_i^1(x_i, k_i) &= \max_{y_i, c_i, \zeta_i} u_i^1(c_i) \quad \text{s.t.} \quad y_i = c_i + x_i + \zeta_i, \quad G_i^1(y_i, k_i) \leq 0, \quad \Lambda_i^1(k_i, \zeta_i) \leq 0 \\
V_i^2(x_i, k_i) &= \max_{y_i, c_i} \beta u_i^2(c_i) \quad \text{s.t.} \quad y_i = c_i + x_i, \quad G_i^2(y_i, k_i) \leq 0
\end{align*}
\]

\(^{21}\)Any equilibrium allocation can be implemented by a continuum of tariff policies that scale up \( 1 + t_T^g(\omega) \) by the same amount for all times \( T \), goods \( g \), and states of the world \( \omega \) (Lerner, 1936; Costinot and Werning, 2019).
Foreign’s geopolitical actions through the time-2 trade quantities it threatens in response to those actions. Although Home can affect time-2 trade quantities indirectly through the impact of time-1 trade taxes on Home capital, it is more efficient to shift them directly using time-2 trade taxes.

4.2 Trade taxes with endogenous Foreign capital

We now turn to the case where Foreign capital is endogenous. In this instance, we show that Home uses its time-1 trade policies to move Foreign’s capital stock in a direction that builds Foreign dependence. Secondarily, we clarify how Home’s time-1 trade policies can move Foreign’s capital stock to compensate for missing international financial markets.

In order to state our characterization of Home trade taxes, we introduce two additional elasticities: Foreign’s \(\text{dynamic terms-of-trade elasticity}\) and its \(\text{dynamic terms-of-trade super-elasticity}\). Both describe the effects of time-1 trade on time-2 outcomes through their impact on Foreign’s capital stock. We capture these effects in a reduced-form way by introducing a function \(\hat{k}_F^1(x_F, \{x_F^2(\omega)\}_{\omega \in \Omega})\) that expresses Foreign capital as a function of its net exports.\(^{22}\)

The dynamic terms-of-trade elasticities capture how trade at time 1 affects Foreign’s terms of trade at time 2 by influencing its capital stock. Formally, we let \(\Phi_g^1(m)\) denote the effect of a change in time-1 exports of \(g\) on the Foreign terms of trade under geopolitical action \(a_F(m)\).

\[
\Phi_g^1(m) \equiv \sum_{d \in D} \frac{x_F^2(m) \cdot \frac{\partial}{\partial x_F^1} \left[ \mu(m)\hat{p}_F^2(x_F^2(m), k_F^1) \right]}{\hat{p}_F^1(x_F^1, k_F^1)} \cdot \hat{k}_F^1(x_F, \{x_F^2(\omega)\}) \cdot \frac{\partial \hat{k}_F^1(x_F^1, \{x_F^2(\omega)\})}{\partial x_F^1} \tag{23}
\]

Similarly, the dynamic terms-of-trade super-elasticities capture how trade at time 1, by influencing Foreign’s capital stock, affects the sensitivity of its terms of trade at time 2 to additional trade. More specifically, the dynamic terms-of-trade super-elasticity corresponding to good \(g\) and action \(a_F(m)\) considers how time-1 trade in \(g\) affects the total terms-of-trade effects that Foreign experiences in moving from trade under \(a_F(m)\) to trade under \(a_F(m + 1)\). Formally, we let \(\Psi_g^1(m)\) denote the effect of a change in time-1 exports of \(g\) on the accumulated terms-of-trade effects Foreign experiences along the path between its net exports under geopolitical actions \(a_F(m)\) and \(a_F(m + 1)\).

\[
\Psi_g^1(m) \equiv \sum_{d \in D} \frac{\partial}{\partial x_F^1} \left[ \int_0^1 \hat{x}_F^2(m)(\zeta) \cdot \frac{\partial}{\partial x_F^1} \left[ \mu(m)\hat{p}_F^2(\hat{x}_F^2(m)(\zeta), k_F^1) \right] \cdot \hat{x}_F^2(m)(\zeta)d\zeta \right] \cdot \hat{k}_F^1(x_F^1, \{x_F^2(\omega)\}) \cdot \frac{\partial \hat{k}_F^1(x_F^1, \{x_F^2(\omega)\})}{\partial x_F^1} \tag{24}
\]

\(^{22}\)We establish the existence of this function in Appendix A.2.
Above, \( \tilde{x}_{F,m}^{2}(\zeta) \) is any smooth path between Foreign’s net exports under geopolitical actions \( a_{F}(m) \) and \( a_{F}(m + 1) \), i.e. between \( x_{F}^{2}(m) \) and \( x_{F}^{2}(m + 1) \).

Our next result characterizes Home’s optimal trade taxes in terms of these elasticities. In order to isolate the geopolitical and missing-market motives for trade policy, we state the result in the case where Home is indifferent between Home and Foreign welfare and in the \( \beta \to 0 \) limit where the conflict period is short. For the general case, see Appendix A.6.

**Proposition 5.** Under Assumption 1 and the assumption that Home is indifferent between Home and Foreign welfare,\(^{23}\) there exist weights \( \lambda_{F}^{2}(m) \) and \( \kappa(m) \) such that Home’s first-period trade taxes and second-period trade threats satisfy

\[
\begin{align*}
t_{H,g}^{1} &= \sum_{m=1}^{M} \lambda_{F}^{2}(m)\Phi_{g}^{1}(m) + \sum_{m=1}^{M-1} \kappa(m)\Psi_{g}^{1}(m) \\
\tau_{H,g}^{2}(m) &= \lambda_{F}^{2}(m)(1 + \Sigma_{g}^{2}(m)) - \Delta\kappa(m)\Sigma_{g}^{2}(m) + o(\beta)
\end{align*}
\]

up to Lerner symmetry, where \( \Delta\kappa(m) \equiv \kappa(m) - \mathbf{1}_{m>1}\mu(m-1)/\mu(m) \cdot \kappa(m-1) \).

The proposition characterizes Home trade taxes in the absence of a redistributive motive across countries. Home’s time-2 taxes are familiar from the case with exogenous Foreign capital covered in Proposition 4. Taxes simply target Foreign’s terms of trade, doing so with an intensity that is higher for geopolitical actions Home wants to discourage more (high \( \Delta\kappa(m) \)) or under which missing markets result in excessive Foreign consumption (high \( \lambda_{F}^{2}(m) \)).\(^{24}\) Home’s time-1 trade taxes are more novel. The result shows that they have two components: A geopolitical component corresponding to the dynamic terms-of-trade super-elasticities \( \Psi_{g}^{1}(m) \) and a missing-markets component corresponding to the dynamic terms-of-trade elasticities \( \Phi_{g}^{1}(m) \).

Although our main focus is on the geopolitical component of Home’s time-1 trade taxes, we briefly discuss the missing-markets component. These reflect the presence of a second market failure in our model in addition to geopolitical externalities: international financial autarky. Home’s time-1 trade taxes seek to compensate for missing financial markets by influencing Foreign’s capital stock, which affects its terms of trade in all time-2 states. To the extent this appreciates Foreign’s terms of trade in states Foreign would like to save for

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\(^{23}\)Formally, consider a bundle of goods \( \{x_{g}^{*1}\} \) with all weakly positive and some strictly positive entries, time-1 trade in the direction of which has a zero first-order impact on Foreign terms of trade and Foreign capital. (Such a bundle generally exists when there are sufficiently many goods.) We assume \( \lambda_{F}^{E} \) is such that, under Home’s optimal policy, Home is indifferent between Home and Foreign consumption of \( x_{g}^{*1} \) at time 1.

\(^{24}\)Time-2 trade taxes also target Foreign capital accumulation to influence Foreign’s decisions between geopolitical actions, but these considerations are second-order in the length of the conflict period.
or insure against, it can improve welfare. While it is interesting, we do not emphasize this channel for two reasons: First, our decision to exclude international financial trade from the model was motivated not by realism but by a desire to abstract from additional considerations related to financial sanctions. For this reason, our policy conclusions concerning international finance should not be taken literally. Second, the missing-markets motive for time-1 trade policy disappears in a standard benchmark case: when Home and Foreign have quasi-linear preferences. In this case, financial autarky is efficient.

We now turn to our main focus: the geopolitical component of Home’s time-1 trade policy, corresponding to the $\Psi^1_g(m)$ terms in (25). Why is it that Foreign’s dynamic terms-of-trade super-elasticities exactly capture Home’s geopolitical motive for trade policy at time 1? Since capital is the only link between times 1 and 2, it is natural that the geopolitical motive operates through the impact of trade on Foreign capital. The question is in what direction Home would like to move Foreign’s capital stock. Intuitively, Home’s goal from a geopolitical stance is to increase the Foreign’s economic incentive of taking geopolitical actions that Home prefers more, such as $a_F(m + 1)$ instead of $a_F(m)$. This incentive equals

$$V^2_F(x^2_F(m + 1), k^1_F) - V^2_F(x^2_F(m), k^1_F) = \int_0^1 \tilde{x}^2_{F,m}''(\zeta) \cdot \frac{d}{dx} \left[ \tilde{p}^2_F(x^2_{F,m}(\zeta), k^1_F) \right] \cdot \tilde{x}^2_{F,m}'(\zeta) d\zeta$$  (26)

where $\tilde{x}^2_{F,m}(\zeta)$ is a smooth path between $x^2_F(m)$ and $x^2_F(m + 1)$. The integrand above represents the sensitivity of the value of Foreign’s net exports—i.e. the sensitivity of Foreign’s terms of trade—to a change in exports $\tilde{x}^2_{F,m}'(\zeta)$. Integrating from $\zeta = 0$ to 1 adds up the full terms-of-trade appreciation Foreign experiences in moving from $x^2_F(m)$ to $x^2_F(m + 1)$. The equality in (26) follows from the fact that the difference in Foreign welfare between two nearby trade vectors that satisfy trade balance is equal to the terms-of-trade effect of moving from one to the other.

So far, we have established that Home seeks to move Foreign capital so as to shift its geopolitical incentives, given by (26). One way to do this is to move Foreign capital in a direction that increases the sensitivity of prices to trade, thereby raising the gains from trade that Foreign experiences between $x^2(m + 1)$ and $x^2(m)$. This is intuitive, as Foreign’s trade elasticity (which is inversely related to the terms-of-trade elasticity) is well understood to play an essential role in determining its gains from trade (Arkolakis et al., 2012). This is exactly the motive reflected in the $\Psi^1_g(m)$ terms in (25).

What is more surprising is that Home does not seek to raise Foreign’s gains from trade indirectly, through the effect of Foreign capital on Foreign’s net exports $x^2_F(m + 1)$ and $x^2_F(m)$ under more- and less-preferred geopolitical actions. Although Home can indeed use time-1 trade policy to affect Foreign’s time-2 net exports through Foreign capital, and although
such changes in net exports do indeed affect Foreign’s economic incentives to take geopolitical actions, it is never optimal for Home to account for these effects in setting its time-1 trade taxes. There are two reasons for this. First, to the extent that the induced changes in time-2 trade are spanned by possible variations in time-2 trade taxes, Home has already determined the level of trade that balances trade-offs between economic and geopolitical welfare. This implies that any geopolitical gains stemming from effects through Foreign capital are, to first order, offset by economic costs. Second, it is also possible for changes in Foreign capital to alter trade in directions that time-2 trade taxes cannot; this occurs when Foreign capital changes move Foreign’s terms of trade. However, Home is indifferent to shifts in terms of trade when it values Foreign consumption equally to its own (except to the extent such shifts compensate for missing financial markets, see above).

Returning to the running example, we now describe Home’s peacetime trade policy in two steps:

1. How would Home like to influence Foreign capital investment?
2. What peacetime trade policies influence Foreign capital investment in the desired way?

**Example.** Home would like to discourage Foreign investment in productive capacity of a downstream (i.e. odd-numbered) good $g$ if Foreign’s consumption utility for $g$ exhibits prudence, i.e. $\frac{\partial^3 u_{cFg}}{\partial c^3} > 0$. By contrast, if Foreign’s consumption utility for $g$ exhibits anti-prudence, i.e. $\frac{\partial^3 u_{cFg}}{\partial c^3} < 0$, Home would like to encourage Foreign investment in productive capacity. And if Foreign’s prudence is 0, Home does not want to change Foreign’s investment.

For typical preferences (e.g. CES), the example implies that Home would like to discourage Foreign investment in *every* type of capital. This contrasts with the naive intuition that Home’s goal is to increase Foreign’s gains from trade and should therefore promote investments better suited for free trade and less suited for autarky. If that were the case, then Home would want to discourage Foreign productive capacity in goods Foreign imports and encourage productive capacity in goods Foreign exports. This strategy is suboptimal because—as described above—Home’s goal is to increase the gains from trade *taking as given* the quantities traded under different geopolitical actions. All that matters, therefore, is how investment influences the responsiveness of prices.

Figure 2 illustrates how, under prudence, reduced investment can affect Foreign gains from trade at fixed quantities by influencing the responsiveness of prices. Both panels depict Foreign’s export supply curve for a particular good at some initial and lower levels of capital $k_F^1$ and $k_F^{1'}$. We shade Foreign’s producer surplus, which accounts for a good’s contribution to Foreign’s gains from trade under restraint relative to aggression. Figure 2a depicts a
good for which Foreign’s utility has zero prudence, while Figure 2b depicts a good for which Foreign’s utility has positive prudence.

In the zero-prudence case, a decrease in productive capacity shifts up Foreign’s export supply curve without changing its slope. Economically, this corresponds to the fact that, without prudence, marginal consumption utility diminishes with quantity at the same rate at all levels of consumption. As a result, Foreign’s gains from trade—holding quantities fixed—are unchanged. In the positive-prudence case, the same decrease in productive capacity not only shifts the export supply curve but also makes it steeper, i.e. less elastic. This reflects that under prudence, marginal consumption utility diminishes faster in quantity at lower consumption levels. By making the exported good scarcer domestically, a reduction in investment therefore makes prices more responsive to marginal changes in the quantity of exports. In this case, Foreign’s gains from trade at fixed quantities rise, increasing Foreign’s economic incentive to choose restraint.

So far, we have emphasized that—for typical preferences exhibiting prudence for all goods—Home would ideally like to discourage all Foreign investment. We next explore for which such goods Home’s motive is the strongest. In general, this problem is difficult, but we can isolate the important parameters in a linear approximation around free trade.

**Example.** Suppose that Home has weak preferences over Foreign’s geopolitical actions and that Home is close to indifferent between consumption at Home and Foreign under both aggression and restraint. All else equal, Home would like to discourage a unit of capital $x_{Fg}$, the coefficient of relative risk aversion in both countries $\sigma_{ig} = -c_{ig} \frac{\sigma_{ig} \mu_{ig}}{\mu_{ig}}$, consumption of
investment in the productive capacity of good \( g \) more than of good \( g' \) if

1. Good \( g \) has a higher relative prudence than good \( g' \), i.e. \( \xi_{Fg} > \xi_{Fg'} \).
2. Foreign’s trade openness to good \( g \) is higher, i.e. \( |x_{Fg}/c_{Fg}| > |x_{Fg'}/c_{Fg'}| \).

The first finding—that Home targets goods with greater relative prudence—is intuitive from Figure 2. Home finds it particularly useful to decrease Foreign investment in the “core goods” for which Foreign has high prudence because they are the goods for which small reductions in investment significantly reduce Foreign elasticities. A natural example is food. For a consumer with a high level of food consumption, marginal utility is insensitive to small changes in that level. For a consumer with a low level of good consumption, marginal utility may be very sensitive to small changes in that level. This provides Home with an incentive to reduce Foreign’s capacity to produce food.

Second, Home would also particularly like to reduce Foreign’s capacity to produce goods for which Foreign’s trade openness \( |x_{Fg}/c_{Fg}| \) is high, on either the import or export side. These are the goods for which Foreign trade and therefore consumption differs significantly between aggression and restraint—i.e. \( |x_{\text{restraint}}| \gg |x_{\text{aggression}}| \) in Figure 2. For such goods, even a small change in the elasticity of Foreign’s net exports supply has a large impact on its gains from trade.

Having explored the ways in which Home would like to influence Foreign investment, we now turn to the question of how Home’s peacetime trade policies can do so.

**Example.** Foreign capital investment in the capacity to produce any downstream good \( g \) is determined by its peacetime exports of that good, \( x_{Fg}^1 \), and the corresponding upstream intermediate good \( g + 1 \), \( x_{F,g+1}^1 \). Furthermore,

\[
\frac{d\tilde{k}_{F,g}}{dx_{Fg}^1} > 0 \quad \text{and} \quad \frac{d\tilde{k}_{F,g+1}}{dx_{F,g+1}^1} < 0.
\]

Quite intuitively, Home can decrease Foreign’s investment in productive capacity for any downstream good using two different trade policies. First, it can decrease Foreign’s net exports of the good itself. This reduces the price of the good in Foreign, decreasing the incentive to invest. Second, it can decrease Foreign’s net imports of the upstream intermediate that good uses, increasing the cost of investing. To the extent Home wants to reduce Foreign’s productive capacity in all goods, as discussed above, this finding has straightforward implications for Home’s peacetime trade policy: For downstream goods, Home should subsidize exports to Foreign and tax imports from Foreign. This echoes Saudi Arabian price cuts

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Home relative to Foreign \( c_{Fg}/c_{Hg} \), Foreign’s relative prudence \( \xi_{Fg} = -c_{Fg} \frac{\partial^2 u_{Fg}}{\partial x_{Fg}^2}/c_{Fg} \), and the marginal product of Foreign capital \( p_{Fg}^2 \frac{\partial^{F_{Fg}}}{\partial k_{Fg}} \).
for oil at the beginning of the US shale boom (The Economist, 2014). For upstream goods, Home should tax exports to Foreign and subsidize exports from Foreign. This echoes recent bans by the US Commerce Department’s Bureau of Industry and Security that prevent US engineers from working in certain Chinese industries (i.e. “exporting engineer services”) (Pao, 2022).

This optimal policy is somewhat subtle. For both upstream and downstream goods, Home encourages trade on one side of the market and discourages trade on the other side of the market. For downstream goods, this creates abundance in Foreign; For upstream goods, it creates scarcity in Foreign. Both of these conditions discourage Foreign capital accumulation and so serve Home’s geopolitical interest.

5 Conclusion

This paper has developed a new framework for studying the interplay of geopolitical conflict and international trade. The premise of our analysis is that a sanctioning country can use peacetime policies to influence capital accumulation both at home and abroad. Doing so can reduce the country’s domestic cost of applying trade sanctions and make foreign countries more susceptible to them. This results in better geopolitical outcomes from the perspective of the sanctioning country.

Within this setting, our analysis has theoretically characterized optimal peacetime capital accumulation and trade policies. From a positive perspective, these results help to explain current and historical policy choices by governments around the globe. From a normative perspective, they suggest ways that these policies could be designed to more efficiently prepare for or prevent geopolitical conflicts.

Future drafts will take this paper’s theoretical insights to the data. In terms of empirics, our results on optimal trade taxes highlight the importance of estimating a new moment: how foreign trade elasticities respond to trade integration. In terms of quantification, it is natural to ask what our model predicts are appropriate geopolitical policies, for example in the context of US relations with China, and how much they affect welfare in each country.

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Appendix

A Main proofs

A.1 Primitive assumptions used below

Our formal results rely on standard neoclassical regularity conditions that we state below.

Assumption 6. For each country \( i \) and time \( T \), utility \( u^T_i \) is strictly increasing, twice continuously differentiable, and strictly concave.

Assumption 7. For each country \( i \) and time \( T \), the goods production possibilities frontier \( G^T_i \) is twice continuously differentiable and weakly convex. We further assume that it is strictly decreasing in \( k \), and at every point \((y, k)\), there is a good \( g \) such that \( \frac{\partial G^T_i}{\partial y_g} > 0 \).

Assumption 8. For each country \( i \), the capital production possibilities frontier \( \Lambda^1_i \) is twice continuously differentiable and weakly convex.

Assumption 9. For each country \( i \) and time \( T \) the Meade utility function \( V^T_i \) is well defined and three times differentiable.

Assumption 10. For each country \( i \) and trade vectors \( x^1_i \), \( \{x^2_2(\omega)\} \), there is a solution to the problem \( \max_{\eta, k} V^1_i(x^1_i + \eta, k^1) + E[V^2_i(x^2_2(\omega), k^1) \text{ such that } \Lambda^1_i(\eta, k^1) \leq 0 \).

Lemma 1. For each country \( i \) and time \( T \) the Meade utility function \( V^T_i \) is well-defined, three times differentiable, strictly concave, has strictly negative derivatives with respect to all goods, and has strictly positive derivatives with respect to all capital types.

Proof. The Meade utility function is

\[
V^T_i(x, k) = \max_y u^T_i(y - x) \quad \text{s.t. } G^T_i(y, k) \leq 0.
\]

By Assumption 9, there is a solution \( y^*_i(x, k) \) and \( V^T_i \) is three times differentiable. Since this is a convex optimization problem with a strictly concave objective function, that solution is unique and is a saddle point of the problem. Since the object and constraints are continuously differentiable, by Kuhn-Tucker there are multiplier \( \lambda^*_i(x, k) \) such that

\[
\frac{\partial u^T_i(y^*_i(x, k) - x)}{\partial c_g} = \lambda^*_i(x, k) \frac{\partial G^T_i(y^*_i(x, k), k)}{\partial y_g}
\]

for each good \( g \), with complementary slackness condition \( \lambda^*_i(x, k) G^T_i(y, k) = 0 \).

We will now show that \( \lambda^*_i(x, k) > 0 \). By Assumption 7, there is a good \( g \) for which \( \frac{\partial G^T_i}{\partial y_g} > 0 \). Further, by Assumption 6, \( \frac{\partial u^T_i(y^*_i(x, k) - x)}{\partial c_g} > 0 \). Therefore, it must be the case that \( \lambda^*_i(x, k) > 0 \).
Finally we can find the derivatives using the envelope theorem. For net exports,
\[
\frac{\partial V_i^T}{\partial x_g} = -\frac{u_i^T(y_i^T(x, k) - x)}{\partial c_g} < 0,
\]
where the inequality follows from Assumption 6. Similarly, by the envelope condition
\[
\frac{\partial V_i^T(x, k)}{\partial k_d} = -\lambda_i^*(x, k) \frac{\partial G_i^T(y_i^T(x, k), k)}{\partial k_d} > 0,
\]
where the inequality follows from Assumption 7.

We finally confirm that \(V_i^T(x, k)\) is strictly concave in \(x\). Suppose that it were not strictly concave so that there are \(x', x''\) and \(\alpha \in [0, 1]\) such that
\[
\alpha u_i^T(\tilde{y}(x') - x') + (1 - \alpha)u_i^T(\tilde{y}(x'') - x'')
\]
where \(\tilde{y}(\cdot)\) is the maximizer and \(x' \equiv \alpha x' + (1 - \alpha)x''\). However, by the strict concavity of \(u_i^T\),
\[
u_i^T(\alpha y(x') + (1 - \alpha)y(x'') - \alpha x' - (1 - \alpha)x'') > \alpha u_i^T(\tilde{y}(x') - x') + (1 - \alpha)u_i^T(\tilde{y}(x'') - x'').
\]
We know that the set of feasible \(y\) implied by \(G_i^T\) is convex, so that \(\alpha y(x') + (1 - \alpha)y(x'')\) is feasible. But then
\[
u_i^T(\alpha y(x') + (1 - \alpha)y(x'') - x) > u_i^T(\tilde{y}(x) - x)
\]
contradicts the fact that \(\tilde{y}(x)\) is optimally chosen. Therefore, \(V_i^T(x, k)\) is strictly concave in \(x\).

\[\square\]

The assumptions above also allow us to characterize time-2 trade in terms of Meade utilities.

**Lemma 2.** For any trade taxes \(t_{H_i}^2\) and capital \(\{k_i^1\}_i\), \(\bar{x}_F^2 = X_F^2(t_{H_i}^2, \{k_i^1\})\) if and only there exist \(\gamma_i > 0\) if and only there exist \(\gamma_i > 0\) for which
\[
V_{F,x}^2(\bar{x}_F^2, k_F^1) \cdot \bar{x}_F^2 = 0 \quad \text{and} \quad 1 + t_{F,g}^2 = \frac{\gamma_H V_{H,x}^2(\bar{x}_F^2, k_H^1)}{\gamma_F V_{x,xy}^2(\bar{x}_F^2, k_F^1)} \quad \text{for all } g \in \mathcal{G} \quad \text{(A1)}
\]

**Proof.** In the forward direction, consider any extension of \(\bar{x}_F^2\) to a full, time-2 equilibrium profile \((\tilde{p}_i^2, \tilde{r}_i^2, \tilde{c}_i^2, \tilde{y}_i^2, \bar{x}_i^2, \tilde{k}_i^{D,2})\) under taxes \(t_{F_i}^2\) and capital \(\{k_i^1\}_i\). By the first welfare theorem conditional on trade and capital, \((\tilde{c}_i^2, \tilde{y}_i^2)\) solves the problem associated with the definition of the Meade utility \(V_i^2\). Combining the FOC for consumption in \(V_i^2\) with the time-2 consumer optimization FOC implies \(\tilde{p}_i^2 x - V_i^2(\tilde{x}_i^2, k_i^1)\). The conditions in (A1) then follow from the Foreign household budget constraint, the world price consistency condition, and international goods market clearing.

In the reverse direction, we extend \(\tilde{x}_F^2\) to a time-2 equilibrium profile \((\tilde{p}_i^2, \tilde{r}_i^2, \tilde{c}_i^2, \tilde{y}_i^2, \bar{x}_i^2, \tilde{k}_i^{D,2})\)
as follows:

\[
\begin{align*}
\tilde{x}_H &= -\tilde{x}_H \\
(c^2_i, y^2_i) &= \arg\max_{c,y} \ u_i^2(c) \quad \text{s.t.} \quad G_i^2(y, k^1_i) \leq 0 \quad \text{and} \quad y = c + x^2_i \\
\hat{p}^2_i &= -\gamma_i V_{i,x}^2(\tilde{x}^2_i, k^1_i) \\
\hat{r}^2_i &= \gamma_i V_{i,k}^2(\tilde{x}^2_i, k^1_i) \\
\hat{k}^{D,2}_i &= \tilde{k}^2_i
\end{align*}
\]

where above we have used the facts that the Meade utility is well-defined and differentiable (from Lemma 1), utility is strictly concave, and the production frontier is weakly convex.

It remains to show that the profile above satisfies the time-2 equilibrium conditions for households, goods producers, world price consistency, trade balance, domestic and international goods market clearing, and capital market clearing. The market clearing conditions are immediate from the construction. Trade balance and world price consistency follow from (A1) and the definition of prices. Household and firm optimization follow from the FOCs for consumption and production, respectively, in (A2), as well as the definition of prices and rental rates.

\[\square\]

A.2 Primal representation lemma

All of our theoretical results rely on a primal representation of the Home planner’s problem. We establish the representation formally below.

Lemma 3. A profile \(\{s^1_H, t^1_H, t^2_H(\omega), \tau^2_H, p^2_1(\omega), r^1_i, r^2_i(\omega), c^1_i, c^2_i(\omega), y^1_i, y^2_i(\omega), k^{D,1}_i, k^{D,2}_i(\omega), l^1_i, k^1_i, k^2_i(\omega), x^1_i, x^2_i(\omega), a_F(\omega)\}_{i=H,F,\omega\in\Omega}\) is an equilibrium if and only if there exist \(\tilde{x}^2_F : A_F \rightarrow \mathbb{R}^\mathcal{G}\) and \(\{\gamma_i > 0\}_{i=H,F}\) for which the following conditions hold for all \(a_F \in A_F, \omega \in \Omega, g \in \mathcal{G}\),

\[
\begin{align*}
\tilde{x}_H &= -\tilde{x}_H \\
(c^2_i, y^2_i) &= \arg\max_{c,y} \ u_i^2(c) \quad \text{s.t.} \quad G_i^2(y, k^1_i) \leq 0 \quad \text{and} \quad y = c + x^2_i \\
\hat{p}^2_i &= -\gamma_i V_{i,x}^2(\tilde{x}^2_i, k^1_i) \\
\hat{r}^2_i &= \gamma_i V_{i,k}^2(\tilde{x}^2_i, k^1_i) \\
\hat{k}^{D,2}_i &= \tilde{k}^2_i
\end{align*}
\]
\[ i \in \{H, F\}, \quad d \in \mathcal{D}: \]

\[
k_F^1 = \tilde{k}_F^1(x_F^1, \{\tilde{x}_F^2(a_F(\omega))\})
\]

\[
V_{F,x}^1(x_F^1, k_F^1) \cdot x_F^1 = 0 \quad \text{and} \quad V_{F,x}^2(\tilde{x}_F^2(a_F), k_F^1) \cdot \tilde{x}_F^2(a_F) = 0
\]

\[
a_F(\omega) \in \arg \max_{a_F \in A_F} V_F^2(\tilde{x}_F^2(a_F), k_F^1) + v_F(a_F, \theta_F(\omega))
\]

\[
\tau_H^2 \in \mathcal{T}_H^2(\{k_F^1\}_{i=H,F})
\]

\[
c_i^1, y_i^1, k_i^1 = \arg \max_{c,y} U_i(c) \quad \text{s.t.} \quad G_i(y, k_i^1) \leq 0, \quad \Lambda_i^1(k_i^1, \nu) \leq 0, \quad y = c + \nu + x_i^1
\]

\[
c_i^2(\omega), y_i^2(\omega) = \arg \max_{c,y} U_i(c) \quad \text{s.t.} \quad G_i(y, k_i^1) \leq 0 \quad \text{and} \quad y = c + x_i^2(\omega)
\]

\[
1 + t_{H,g}^1 = \frac{\gamma_H V_{H,x_g}^1(x_F^1, k_F^1)}{\gamma_F V_{F,x_g}^1(x_F^1, k_F^1)} \quad \text{and} \quad 1 + \tau_{H,g}^2(a_F) = \frac{\gamma_H V_{H,x_g}^2(-\tilde{x}_F^2(a_F), k_F^1)}{\gamma_F V_{F,x_g}^2(-\tilde{x}_F^2(a_F), k_F^1)}
\]

\[
p_i^1 = -\gamma_i V_{i,x}^1(x_i^1, k_i^1) \quad \text{and} \quad r_i^1 = -\frac{G_{i,k}(y_i^1, k_i^1)}{G_{i,y}(y_i^1, k_i^1)} p_i^1
\]

\[
p_i^2(\omega) = -\gamma_i V_{i,x}^2(x_i^2(\omega), k_i^1) \quad \text{and} \quad r_i^2(\omega) = \gamma_i V_{i,k}^2(x_i^2(\omega), k_i^1)
\]

\[
t_{H}^2(\omega) = \tau_{H}^2(a_F(\omega))
\]

\[
1 + s_{H,d}^1 = \frac{\Lambda_{H,k_d}^1(x_H^1, k_H^1)}{V_{H,k_d}^1(x_H^1, k_H^1) + \mathbb{E}[V_{H,k_d}^2(x_H^2(\omega), k_H^1)]}
\]

\[
k_i^{D,1} = k_i^1 \quad \text{and} \quad k_i^{D,2}(\omega) = k_i^2(\omega) = k_i^1
\]

\[
x_F^2(\omega) = \mathcal{X}_F^2(t_{H}^2(\omega), \{k_i^1\})
\]

\[
x_H^1 = -x_F^1 \quad \text{and} \quad x_H^2(\omega) = -x_F^2(\omega)
\]

(A3)

where \(\tilde{k}_F^1(x_F^1, \{\tilde{x}_F^2(\omega)\})\) is the (we show) unique level of Foreign capital consistent with time-1 net exports \(x_F^1\) and time-2 net exports \(x_F^2(\omega)\).\(^{26}\)

\[ Proof. \] We begin by restating the full set of equilibrium conditions: An equilibrium is a profile \(\{s_{H}, t_{H}^1, t_{H}^2(\omega), \tau_{H}, p_{H}^1, p_{H}^2(\omega), r_{H}^1, r_{H}^2(\omega), c_{i}^1, c_{i}^2(\omega), y_{i}^1, y_{i}^2(\omega), k_{i}^{D,1}, k_{i}^{D,2}(\omega), \nu_{i}^1, k_{i}^1, k_{i}^2(\omega), x_{H}^1, x_{H}^2(\omega),\]

\(^{26}\)Our subsequent analysis additionally assumes \(\tilde{k}_F^1(x_F^1, \{\tilde{x}_F^2(\omega)\})\) is differentiable.
\( a_F(\omega) \) that satisfies
\[
\begin{align*}
\{c^1_i, (c^2_i(\omega))\} \omega \in 
& \arg\max_{c^1_i, (c^2_i(\omega))} u_i^1(c^1) + \beta \mathbb{E}[u_i^2(c^2(\omega))] \\
& \text{s.t. } p_i^1 \cdot c^1 + \mathbb{E}[p_i^2(\omega) \cdot c^2(\omega)] \leq \pi_i + R_i \\
& \quad \pi_i = p_i^1 \cdot (y_i - u_i^1) + s_i^1 r_i^1 \cdot k_i^1 + \mathbb{E}[p_i^2(\omega) \cdot y_i^2(\omega)] \\
& \quad R_i = \frac{p_i^1 t_i^1}{1 + t_i^1} \cdot x_i^1 - s_i^1 r_i^1 \cdot k_i^1 + \mathbb{E} \left[ \frac{p_i^2 t_i^2}{1 + t_i^2} \cdot x_i^1 \right] \\
& y_i^1, k_i^{D,1} \in \arg\max_{y, k} \; p_i^1 \cdot y - r_i^1 \cdot k \quad \text{s.t. } G_i^1(y, k) \leq 0 \\
& y_i^2(\omega), k_i^{D,2}(\omega) \in \arg\max_{y, k} \; p_i^2(\omega) \cdot y - r_i^2(\omega) \cdot k \quad \text{s.t. } G_i^2(y, k) \leq 0 \\
& \{t_i^1, k_i^1, \{k_i^2(\omega)\}\} \omega \in \arg\max_{t_i^1, k_i^1, \{k_i^2(\omega)\}} \; \mathbb{E}[r_i^2(\omega) \cdot k_i^2(\omega)] \\
& \quad \text{s.t. } \Lambda_i^1(k_1^1, t_i^1) \leq 0 \\
& \quad \forall \omega \in \Omega, \quad k_i^2(\omega) = k_i^1 \\
& \quad p_i^1 \cdot x_i^1 = 0 \quad \text{and} \quad p_i^2(\omega) \cdot x_i^2(\omega) = 0 \\
& \quad k_i^1 = k_i^{D,1} \quad \text{and} \quad k_i^2(\omega) = k_i^{D,2}(\omega) \\
& \quad y_i^1 = c_i^1 + t_i^1 + x_i^1 \quad \text{and} \quad y_i^2(\omega) = c_i^2(\omega) + x_i^2(\omega) \\
& \quad x_i^1 + x_i^1 = 0 \quad \text{and} \quad x_i^2(\omega) + x_i^2(\omega) = 0 \\
& a_F(\omega) \in \arg\max_{a_F} \; u_F^2 \left( C_F^2(t_i^1, \{k_i^1(\omega)\}_{i=H,F}) \right) + V_F(a_F, \theta_F(\omega)) \\
& \quad \text{s.t. } t_i^2 = \tau_i^2(a_F) \\
& \quad t_i^2(\omega) = \tau_i^2(a_F(\omega)) \\
& \quad \tau_i^2 \in \mathcal{T}^2_i(\{k_i^1(\omega)\}_{i=H,F})
\end{align*}
\]

By the first and second welfare theorems conditional on trade and capital, the condition from capital producer optimization that \( k_i^2(\omega) = k_i^1 \), and the strict concavity of \( u_i^2 \) and the weak convexity of \( G_i^2 \) and \( \Lambda_i^2 \), the equilibrium conditions for consumption, production, and goods market clearing are equivalent to
\[
\begin{align*}
& \{c^1_i, y_i^1, t_i^1\} = \arg\max_{c, y, t} \; u_i^1(c) \quad \text{s.t. } G_i^1(y, k_i^1) \leq 0, \quad \Lambda_i^1(k_i^1, t_i^1) \leq 0, \quad y = c + t_i^1 + x_i^1 \\
& \{c^2_i(\omega), y_i^2(\omega)\} = \arg\max_{c, y} \; u_i^2(c) \quad \text{s.t. } G_i^2(y, k_i^1) \leq 0, \quad y = c + x_i^2(\omega) \\
\end{align*}
\]

Also by the first welfare theorem conditional on trade and capital, as well as the condition from capital producer optimization that \( k_i^2(\omega) = k_i^1 \), we have
\[
\begin{align*}
& u_F^2 \left( C_F^2(t_i^1, \{k_i^2(\omega)\}_{i=H,F}) \right) = V_F^2 \left( \mathcal{X}_F^2(t_i^1, \{k_i^1(\omega)\}_{i=H,F}), k_i^F \right) \\
\end{align*}
\]

Additionally, from the first-order conditions associated with the of the Meade utility
function; household optimization, and capital producer optimization; and the fact that utility is strictly increasing, there exist constants \( \gamma_i > 0 \) for which

\[
p^1_i = -\gamma_i V^1_{i,x}(x^1_i, k^1_i) \quad \text{and} \quad r^1_i = -\frac{G^1_{i,k}(y^1_i, k^1_i)}{G^1_{i,y}(y^1_i, k^1_i)} p^1_i \tag{A7}
\]

\[
p^2_\omega = -\gamma_i V^2_{i,x}(x^2_\omega(\omega), k^1_i) \quad \text{and} \quad r^2_\omega(\omega) = \gamma_i V^2_{i,k}(x^2_\omega(\omega), k^1_i)
\]

Using these observations, we restate the equilibrium conditions as: An equilibrium is a profile \( \{s^1_H, t^1_H, t^2_H(\omega), \tau^2_H, p^1_i, p^2_\omega, r^1_i, r^2_\omega(\omega), c^1, c^2_\omega(\omega), y^1_i, y^2_\omega(\omega), k^1_D, k^1_D^2(\omega), l^1_i, k^1_i, k^2_\omega(\omega), x^1_i, x^2_\omega(\omega), a_F(\omega)\}_{i=H,F,\omega}\in\Omega} \) for which there exist \( \{\gamma_i > 0\}_{i=H,F} \) such that

\[
\begin{align*}
\left\{ i^1_i, k^1_i, \{k^2_\omega(\omega)\}_{\omega\in\Omega} \right\} & \in \arg\max_{i^1_i, k^1_i, \{k^2_\omega(\omega)\}_{\omega\in\Omega}} V^1_{i,k}(x^1_i, k^1_i)(1 + s^1_i) \cdot k^1 + V^1_{i,x}(x^1_i, k^1_i) \cdot l^1 + \mathbb{E}[V^2_{i,k}(x^2_\omega(\omega), k^1_i) \cdot k^2_\omega(\omega)] \\
\text{s.t.} \quad & \Lambda^1_i(k^1_i, l^1) \leq 0 \\
& \forall \omega \in \Omega, \quad k^2_\omega(\omega) = k^1 \\
V^1_{F,x}(x^1_F, k^1_F) \cdot x^1_F = 0 & \quad \text{and} \quad V^2_{F,x}(x^2_\omega(\omega), k^1_F) \cdot x^2_\omega(\omega) = 0 \\
x^1_H + x^1_F = 0 & \quad \text{and} \quad x^2_\omega(\omega) + x^2_F(\omega) = 0 \\
a_F(\omega) & \in \arg\max_{a_F \in A_F} V^2_F(\mathcal{A}^2_F(\tau^2_H(a_F), \{k^1_i\}_{i=H,F}), k^1_F) + v_F(a_F, \theta_F(\omega)) \\
\tau^2_H & \in \mathcal{T}^2_H(\{k^1_i\}_{i=H,F}) \\
c^1, y^1_i, l^1_i & = \arg\max_{c, y, l} u^1_i(c) \quad \text{s.t.} \quad G^1_i(y, k^1_i) \leq 0, \quad \Lambda^1_i(k^1_i, l) \leq 0, \quad y = c + l + x^1_i \\
c^2_\omega(\omega), y^2_\omega(\omega) & = \arg\max_{c, y} u^2_i(c) \quad \text{s.t.} \quad G^2_i(y, k^1_i) \leq 0 \quad \text{and} \quad y = c + x^2_\omega(\omega)
\end{align*}
\]

\[
1 + t^1_{H,g} = \frac{\gamma_H V^1_{H,x,g}(x^1_H, k^1_H)}{\gamma_F V^1_{F,x,g}(x^1_F, k^1_F)} \quad \text{and} \quad 1 + t^2_{H,g}(\omega) = \frac{\gamma_H V^2_{H,x,g}(x^2_H(\omega), k^1_H)}{\gamma_F V^2_{F,x,g}(x^2_\omega(\omega), k^1_F)}
\]

\[
p^1_i = -\gamma_i V^1_{i,x}(x^1_i, k^1_i) \quad \text{and} \quad r^1_i = -\frac{G^1_{i,k}(y^1_i, k^1_i)}{G^1_{i,y}(y^1_i, k^1_i)} p^1_i \\
p^2_\omega = -\gamma_i V^2_{i,x}(x^2_\omega(\omega), k^1_i) \quad \text{and} \quad r^2_\omega(\omega) = \gamma_i V^2_{i,k}(x^2_\omega(\omega), k^1_i) \\
t^2_H(\omega) = \tau^2_H(a_F(\omega)) \quad \text{and} \quad \quad \quad \quad k^1_D = k^1_i \quad \text{and} \quad k^1_D^2(\omega) = k^2_\omega(\omega)
\]

where above we have used the fact (from Lemma 1) that the derivatives of \( V^T_i \) are non-zero to rewrite the world-price consistency conditions.

We next consider the problem of capital producers. The convexity of \( \Lambda^1_i \) and strict concavity of \( V^T_i \) imply capital producer optimization is equivalent to:

\[
\begin{align*}
\left\{ i^1_i, k^1_i \right\} & \in \arg\max_{i^1_i, k^1_i} \left( 1 + s^1_i \right) V^1_i(x^1_i - l^1_i + c^1, k^1) + \mathbb{E}[V^2_i(x^2_\omega(\omega), k^1)] \\
\text{s.t.} \quad & \Lambda^1_i(k^1_i, l^1_i) \leq 0
\end{align*}
\]
plus the condition $k_i^2(\omega) = k_i^1$ for all $\omega \in \Omega$. Note that (A9) is equivalent to the conditions

$$
k_i^1 \in \arg\max_{k_i^1} \max_{\ell_i^1} (1 + s_i^1) V_i^1(x_i^1 - \ell_i^1 + \ell_i^1, k_i^1) + \mathbb{E}[V_i^2(x_i^2(\omega), k_i^1)] \quad \text{s.t.} \quad \Lambda_i^1(k_i^1, \ell_i^1) \leq 0
$$

$$
\ell_i^1 \in \arg\max_{\ell_i^1} V_i^1(x_i^1 - \ell_i^1 + \ell_i^1, k_i^1) \quad \text{s.t.} \quad \Lambda_i^1(k_i^1, \ell_i^1) \leq 0
$$

(A10)

As the second condition in (A10) is already included in (A8), (A8) is equivalent to a version of the equilibrium conditions that replaces the capital producer optimization condition in (A8) with the first condition in (A10). Before making this substitution, we note that the first uniquely determines $k_i^1$ as a function of $x_i^1$ and $t_i^1$, since a solution to that problem exists by Assumption 10 and any such solution is unique by the strict concavity of $V_i^T$ and the weak convexity of $\Lambda_i^1$. We therefore define $\tilde{k}_F^2(x_i^1, \{x_i^2(\omega)\})$ as the unique solution to

$$
\max_{k_i^1} \max_{\ell_i^1} (1 + s_i^1) V_i^1(x_i^1 - \ell_i^1 + \ell_i^1, k_i^1) + \mathbb{E}[V_i^2(x_i^2(\omega), k_i^1)] \quad \text{s.t.} \quad \Lambda_i^1(k_i^1, \ell_i^1) \leq 0 \quad (A11)
$$

We additionally note that—by the concavities and convexities noted above and the fact from Lemma 1 that $V_i^T$’s derivatives are non-zero—the first condition in (A10) is, for Home, equivalent to the FOC

$$
1 + s_{H,d}^1 = \frac{\Lambda_{H,k_H}(x_H^1, k_H^1)}{V_{H,k_H}(x_H^1, k_H^1) + \mathbb{E}[V_{H,k_H}(x_H^2(\omega), k_H^1)]} \Lambda_{H,x_{g_1}}(x_H^1, k_H^1) \quad (A12)
$$

for any good $g_1 \in G$ (the RHS is constant across goods by the condition for investment in (A8)).

Combining these observations, we restate the equilibrium conditions as: An equilibrium is a profile $\{s_H^1, t_H^1, \tau_H^2, p_H^1, p_H^2(\omega), r_i^1, r_i^2(\omega), c_i^1, c_i^2(\omega), y_i^1, y_i^2(\omega), k_i^{D,1}, k_i^{D,2}(\omega), \ell_i^1, k_i^1, k_i^2(\omega), x_i^1, \ldots\}$,
\[ x_i^2(\omega), a_F(\omega) \}_{i=H,F,\omega \in \Omega} \text{ for which there exist } \{ \gamma_i > 0 \}_{i=H,F} \text{ such that} \]

\[ k_F^i = \tilde{k}_F(x_F^i, \{ x_F^2(\omega) \}) \]
\[ V_{F,x}^1(x_F^i, k_F^i) \cdot x_F^i = 0 \quad \text{and} \quad V_{F,x}^2(x_F^2(\omega), k_F^i) \cdot x_F^2(\omega) = 0 \]
\[ a_F(\omega) = \arg \max_{a_F \in A_F} V_F^2(A_F^2(\tau_H^2(a_F), \{ k_i^1 \}_{i=H,F}), k_F^i) + v_F(a_F, \Theta_F(\omega)) \]
\[ \tau_H^2 \in \mathcal{T}_H^2(\{ k_i^1 \}_{i=H,F}) \]
\[ c_i^1, y_i^1, q_i^1 = \arg \max_{c,y,q} u_i^1(c) \quad \text{s.t.} \quad G_i^1(y, k_i^1) \leq 0, \quad \Lambda_i^1(k_i^1, t) \leq 0, \quad y = c + t^1 \]
\[ c_i^2(\omega), y_i^2(\omega) = \arg \max_{c,y} u_i^2(c) \quad \text{s.t.} \quad G_i^2(y, k_i^1) \leq 0 \quad \text{and} \quad y = c + x_i^2(\omega) \]
\[ 1 + t_{H,g}^1 = \frac{\gamma_H V_{H,x}^1(x_H^i, k_H^1)}{\gamma_F V_{F,x}^1(x_F^i, k_F^i)} \quad \text{and} \quad 1 + t_{H,g}^2(\omega) = \frac{\gamma_H V_{H,x}^2(x_H^2(\omega), k_H^1)}{\gamma_F V_{F,x}^2(x_F^2(\omega), k_{F}^1)} \]
\[ p_i^1 = -\gamma_i V_{i,x}^1(x_i^1, k_i^1) \quad \text{and} \quad r_i^1 = -\frac{G_i^1(y_i^1, k_i^1)}{G_i^1(y_i^1, k_i^1)} p_i^1 \]
\[ p_i^2(\omega) = -\gamma_i V_{i,x}^2(x_i^2(\omega), k_i^1) \quad \text{and} \quad r_i^2(\omega) = \gamma_i V_{i,k}^2(x_i^2(\omega), k_i^1) \]
\[ t_{H^2}^i(\omega) = \tau_H^2(a_F(\omega)) \]
\[ 1 + s_{H,d}^1 = \frac{\Lambda_{H,k_d}(x_H^1, k_H^1)}{V_{H,k_d}^1(x_H^1, k_H^1) + \mathbb{E}[V_{H,k_d}^2(x_H^2(\omega), k_H^1)]} \]
\[ k_i^{D,1} = k_i^1 \quad \text{and} \quad k_i^{D,2}(\omega) = k_i^2(\omega) = k_i^1 \]
\[ x_i^1 = -x_F^1 \quad \text{and} \quad x_i^2(\omega) = -x_F^2(\omega) \]

(A13)

Next, note that by Lemma 2, it is equivalent to replace \( X_F^2(\tau_H^2(a_F), \{ k_i^1 \}_{i=H,F}) \) with some net export vector \( \tilde{x}^2_F(a_F) \) provided we add to the equilibrium conditions the requirements that, for some \( \{ \gamma_i > 0 \}_{i=H,F} \),

\[ V_{F,x}^2(\tilde{x}_F^i, k_F^i) \cdot \tilde{x}_F^i(a_F) = 0, \quad 1 + \tau_{F,g}^2(\omega) = \frac{\gamma_H V_{H,x}^2(-\tilde{x}_F^2(a_F), k_H^1)}{\gamma_F V_{F,x}^2(\tilde{x}_F^2(a_F), k_F^1)} \quad \text{for all} \quad g \in \mathcal{G} \quad \text{(A14)} \]

We use this observation to restate the equilibrium conditions yet again: An equilibrium is a profile \( \{ s_H^1, t_H^1, \tau_H^2, \omega, \tau_H^2, p_i^1, p_i^2(\omega), r_i^1, r_i^2(\omega), c_i^1, c_i^2(\omega), y_i^1, y_i^2(\omega), k_i^{D,1}, k_i^{D,2}(\omega), l_i^1, k_i^1, k_i^2(\omega), x_i^1, \]
\[ x_i^2(\omega), a_F(\omega) \}_{i=H,F,\omega \in \Omega} \text{ for which there exist } \tilde{x}_F^i : \mathcal{A}_F \rightarrow \mathbb{R}^g \text{ and } \{ \gamma_i > 0 \}_{i=H,F} \text{ such that} \]
Equation A3 holds. This completes the proof.

\[ \square \]

### A.3 Proof of Proposition 1

The planner’s problem (stated formally in Equation 8) is to policies \( s_H^1, t_H^1, \tau_H^2 \) and equilibrium given policies \( \{ s_H^1, t_H^1, \tau_H^2, \omega, \tau_H^2, p_i^1, p_i^2(\omega), r_i^1, r_i^2(\omega), c_i^1, c_i^2(\omega), y_i^1, y_i^2(\omega), k_i^{D,1}, k_i^{D,2}(\omega), l_i^1, \)
\[ k_1^1, k_1^2(\omega), x_1^1, x_1^2(\omega), a_F(\omega) \}_{i=H,F,\omega \in \Omega} \] to maximize the objective

\[ u_H^1(c_H^1) + \beta \mathbb{E}\left[ u_H^2(c_H^2(\omega)) + v_H(a_F(\omega)) \right] + \lambda^E u_F^1(c_F^1) + \beta \mathbb{E}\left[ \lambda^E u_F^2(c_F^2(\omega)) + \lambda^G v_F(a_F(\omega), \theta_F(\omega)) \right] \] \quad (A15)

Note that, by the first welfare theorem conditional on trade and capital, we may replace \( u_H^1(c_H^1) \) and \( u_F^1(c_F^1(\omega)) \) with \( V_H^1(x_1^1, k_1^1) \) and \( \beta V_F^2(x_1^2(\omega), k_1^2(\omega)) \), respectively. By capital feasibility, capital market clearing, and international goods market clearing, we may further replace \( k_1^2(\omega) \) with \( k_1^1 \), \( x_1^1 \) with \(-x_1^1\), and \( x_1^2(\omega) \) with \(-x_1^2(\omega)\). Using this observation to rewrite the planner’s objective and using Lemma 3 to rewrite the equilibrium conditions, the planner’s problem can be stated as follows.

\[
\max_{\{x_1^1, x_1^2, x_1^2(a_F)\}} \quad V_H^1(-x_1^1, k_1^1) + \mathbb{E}\left[ V_H^2(-x_1^2(\omega), k_1^1) + v_H(a_F(\omega)) \right] \\
+ \lambda^E V_F^1(x_1^1, k_1^1) + \mathbb{E}\left[ \lambda^E V_F^2(x_1^2(\omega), k_1^1) + \lambda^G v_F(a_F(\omega), \theta_F(\omega)) \right] \\
\text{s.t. Equation (A3) holds} \quad (A16)
\]

Note that above, we may additionally replace \( x_1^2(\omega) \) with \( \tilde{x}_F^2(\omega, a_F(\omega)) \) since the conditions in (A3) ensure that both equal \( \lambda^F_2(\tilde{t}_F^2, \{k_1^1\}) \).

Studying the equations in (A3) reveals that the first four equations—as well as the planner’s objective as stated above—do not depend on any variables determined in any of the subsequent equations. This observation hinges on the assumption that the set of feasible tariff threats \( T_F^2 \) is unconstrained. The planner’s problem therefore simplifies to

\[
\max_{\{k_1^1, x_1^1, \tilde{x}_F^2(a_F), a_F(\omega)\}} \quad V_H^1(-x_1^1, k_1^1) + \mathbb{E}\left[ V_H^2(-\tilde{x}_F^2(a_F(\omega)), k_1^1) + v_H(a_F(\omega)) \right] \\
+ \lambda^E V_F^1(x_1^1, k_1^1) + \mathbb{E}\left[ \lambda^E V_F^2(\tilde{x}_F^2(a_F(\omega)), k_1^1) + \lambda^G v_F(a_F(\omega), \theta_F(\omega)) \right] \\
\text{s.t.} \quad k_1^1 = \tilde{k}_F^1(x_1^1, \{\tilde{x}_F^2(a_F(\omega))\}) \\
V_F^1(x_1^1, k_1^1) \cdot x_1^1 = 0 \\
V_F^2(x_1^1, k_1^1) \cdot \tilde{x}_F^2(a_F(\omega)) = 0 \\
a_F(\omega) \in \arg \max_{a_F \in A_F} V_F^2(\tilde{x}_F^2(a_F), k_1^1) + v_F(a_F, \theta_F(\omega)) \quad (A17)
\]

While extraneous to the planner’s problem as formulated above, the equilibrium conditions not used above can be used to connect the problem’s solution to the necessary values of other equilibrium quantities in the planner’s solution.

Note that Home capital \( k_1^1 \) only appears in (A17) through the objective, implying the necessary first-order condition:

\[ V_{H,k}^1(x_1^1, k_1^1) + \mathbb{E}\left[ V_{H,k}^2(x_1^2(\omega), k_1^1) \right] = 0 \] \quad (A18)
where here we have again applied the observation above that \( x_H^1 = -x_F^1 \) and \( x_H^2(\omega) = \widetilde{x}_F^2(a_F(\omega)) \). It remains to relate the terms in (A18) to interest rates. We have already shown in the proof of Lemma 3 that in any equilibrium there exists \( \gamma_H > 0 \) such that \( \gamma_H V_{H,k}^2(x_F^1(\omega), k_F^1) = r_H^2(\omega) \). Combining the first-order conditions for firms and households with the definition of the Meade utility function implies that, for the same \( \gamma_H \), we have \( \gamma_H V_{H,k}^2(x_F^1(\omega), k_F^1) = -(r_H^1 s_H^1 + E[r_H^2(\omega)]) \). Combining these observations, (A18) implies

\[
-(r_H^1 s_H^1 + E[r_H^2(\omega)]) + E[r_H^2(\omega)] = -r_H^1 s_H^1 = 0
\]

i.e. Home’s capital subsidies are zero.

### A.4 Proof of Proposition 2

We begin as in the proof of Proposition 1, but now replacing the constraint \( \tau_H^2 \in \mathcal{T}_H^2(\{k_i^1\}_{i=H,F}) \) on the planning problem in (A16) using Assumption 2. Under this assumption, \( \tau_H^2 \in \mathcal{T}_H^2(\{k_i^1\}_{i=H,F}) \) is equivalent to

\[
\max_{a_F \in A_F} u_H^2(C_H^2(\tau_H^2(a_F), \{k_i^1\})) \; - \; \min_{a_F \in A_F} u_H^2(C_H^2(\tau_H^2(a_F), \{k_i^1\})) \leq \delta
\]

or, using the observation from the proof of Lemma 3 that \( u_H^2(C_H^2(\tau_H^2(a_F), \{k_i^1\})) = V_H^2(\bar{x}_F^2(a_F), k_H^1) \),

\[
\max_{a_F \in A_F} V_H^2(\bar{x}_F^2(a_F), k_H^1) \; - \; \min_{a_F \in A_F} V_H^2(\bar{x}_F^2(a_F), k_H^1) \leq \delta. \tag{A21}
\]

We now simplify the planner’s problem as in Proposition 1, noting that the equation above only depends on variables already included in unconstrained primal problem in (A17). The constrained primal problem under Assumption 2 therefore takes the form

\[
\max_{\{k_i^1, x_F^1, x_F^2(a_F, \omega)\}} \left[ V_H^1(-x_F^1, k_H^1) + E[V_H^2(-\bar{x}_F^2(a_F(\omega)), k_H^1) + v_H(a_F(\omega))] + \lambda^F V_F^1(x_F^1, k_F^1) + \lambda^E V_F^2(\bar{x}_F^2(a_F(\omega), k_F^1) + \lambda^G v_F(a_F(\omega), \theta_F(\omega)) \right]
\]

s.t.

\[
\begin{align*}
&k_F^1 = \bar{k}_F^1(x_F^1, \{\bar{x}_F^2(a_F(\omega))\}) \\
&V_F^1(x_F^1, k_F^1) \cdot x_F^1 = 0 \\
&V_F^2(\bar{x}_F^2(a_F), k_F^1) \cdot \bar{x}_F^2(a_F) = 0 \\
a_F(\omega) \in \arg\max_{a_F \in A_F} V_F^2(\bar{x}_F^2(a_F), k_F^1) + v_F(a_F, \theta_F(\omega)) \\
&\max_{a_F \in A_F} V_H^2(\bar{x}_F^2(a_F), k_H^1) \; - \; \min_{a_F \in A_F} V_H^2(\bar{x}_F^2(a_F), k_H^1) \leq \delta
\end{align*}
\]

(A22)

As in the proof of Proposition 1, we now consider the planner’s necessary first-order condition with respect to Home capital, letting \( \kappa > 0 \) denote the Lagrange multiplier on the
credibility constraint.\(^{27}\)

\[
\bar{\theta} = V_{H,k}^1(x_H^1, k_F^1) + \mathbb{E}[V_{H,k}^2(x_H^2(\omega), k_F^1)] - \kappa \left[ V_{R,H}^2(\tilde{x}_H^2(\bar{\alpha}_H), k_F^1) - V_{H}^2(\tilde{x}_H^2(\bar{\alpha}_H), k_F^1) \right]
\]

By the same steps as in the proof of Proposition 1, this implies

\[
\bar{\theta} = - (r_H^1 s_H^1 + \mathbb{E}[r_H^2(\omega)]) + \mathbb{E}[r_H^2(\omega)] - \kappa \left[ r_H^2(\bar{\omega}) - r_H^2(\omega) \right]
\]

(A24)

where \(\bar{\omega}\) and \(\bar{\omega}\) are some states in which \(a_F(\bar{\omega}) = \bar{\alpha}_F\) and \(a_F(\bar{\omega}) = \bar{\alpha}_F\). The proposition follows from defining \(\tilde{\mu}\) to be some signed measure that—relative to the true probability measure \(\mu\)—assigns additional weight \(\kappa\) to state \(\bar{\omega}\), taking this weight from state \(\bar{\omega}\).

A.5 Proof of Proposition 3

We begin as in the proof of Proposition 1, but now replacing the constraint \(\tau_H^2 \in \mathcal{T}_H^2 \{k_i\}_{i=H,F}\) on the planning problem in (A16) using Assumption 3. Under this assumption, \(\tau_H^2 \in \mathcal{T}_H^2 \{k_i\}_{i=H,F}\) is equivalent to

\[
\forall a_F \in \mathcal{A}_F \quad \text{s.t.} \quad v_H(a_F) \geq \bar{v}_H, \quad \tau_H^2(a_F) = \bar{\theta}.
\]

(A25)

As in the proof of Proposition 1, we focus on a subset of equilibrium equations in the general planner’s problem of Equation A16, including the objective, that do not depend on any variables determined in other equilibrium equations. The only difference relative to Proposition 1 is that the variables used in these conditions now include \(\gamma_H\) and \(\gamma_F\), and, together with the world-price-consistency condition, (A25) forms an additional constraint these variables must satisfy. This gives us the problem:

\[
\max_{\{k_1, x_F, \tilde{x}_F(a_F)\}} \quad V_H^1(-\tilde{x}_F, k_H^1) + \mathbb{E}\left[ V_H^2(-\tilde{x}_F(a_F(\omega)), k_F^1) + v_H(a_F(\omega)) \right] + \lambda^E V_F^1(x_F^1, k_F^1) + \mathbb{E}\left[ \lambda^E V_F^2(\tilde{x}_F(a_F(\omega)), k_F^1) + \lambda^G v_F(a_F(\omega), \theta_F(\omega)) \right]
\]

s.t. \(k_F^1 = \tilde{k}_F^1(x_F^1, \tilde{x}_F(a_F(\omega)))\)

\(V_{F,x}(x_F^1, k_F^1) \cdot x_F = 0\)

\(V_{F,x}(\tilde{x}_F(a_F), k_F^1) \cdot \tilde{x}_F(a_F) = 0\)

\(a_F(\omega) \in \arg \max_{a_F \in \mathcal{A}_F} V_F^2(\tilde{x}_F(a_F), k_F^1) + v_F(a_F, \theta_F(\omega))\)

\(1 = \frac{\gamma_H V_H^1(-\tilde{x}_F(a_F), k_H^1)}{\gamma_F V_F^2(\tilde{x}_F(a_F), k_F^1)} \quad \forall a_F \in \mathcal{A}_F \quad \text{s.t.} \quad v_H(a_F) \geq \bar{v}_H
\]

(A26)

---

\(^{27}\)We assume this multiplier exists.
Recall that by Lemma 2, the two conditions

\[ V_{F,x}^2(\vec{x}_F(a_F), k_F^1) \cdot \vec{x}_F(a_F) = 0 \quad \text{and} \quad 1 = \frac{\gamma_H V_{H,x_0}^2(\vec{x}_F(a_F), k_H^1)}{\gamma_F V_{F,x_0}^2(\vec{x}_F(a_F), k_F^1)} \]  

(A27)

are equivalent to \( \vec{x}_F^2(a_F) = \mathcal{X}_F^2(\bar{0}, \{k^1_F\}) \). Restating the planner’s problem using this equivalence and defining \( \bar{A}_F = \{a_F \in A_F \mid v_H(a_F) < \bar{v}_H\} \) and \( \vec{x}_F^{2,FT}({k^1_F}) = \mathcal{X}_F^2(\bar{0}, \{k^1_F\}) \), we have

\[
\max_{\{k^1_F, x_F, \vec{x}_F(a_F), a_F(\omega), \gamma_i\}} \quad V_H^1(-x_F^1, k_H^1) + \mathbb{E} \left[ V_H^2(-\vec{x}_F(a_F(\omega)), k_H^1) + v_H(a_F(\omega)) \right]
\]

\[ + \lambda^E V_F^1(x_F^1, k_F^1) + \mathbb{E} \left[ \lambda^E V_F^2(\vec{x}_F(a_F(\omega)), k_F^1) + \lambda^G v_F(a_F(\omega), \theta_F(\omega)) \right] \]

s.t. \( k_F^1 = \bar{k}_F^1(x_F^1, \{\vec{x}_F(a_F(\omega))\}) \)

\[ V_{F,x}^1(x_F^1, k_F^1) \cdot x_F^1 = 0 \]

\[ V_{F,x}^2(\vec{x}_F(a_F), k_F^1) \cdot \vec{x}_F(a_F) = 0 \]

\[ a_F(\omega) \in \arg \max_{a_F \in A_F} V_F^2(\vec{x}_F(a_F), k_F^1) + v_F(a_F, \theta_F(\omega)) \]

\[ \vec{x}_F(a_F) = \vec{x}_F^{2,FT}({k^1_F}) \quad \forall a_F \notin \bar{A}_F \]  

(A28)

This implies the following necessary first-order condition for Home capital:

\[
\vec{0} = V_{H,k}^1(x_H^1, k_H^1) + \mathbb{E} \left[ V_{H,k}^2(x_H^2(\omega), k_H^1) \right] + \sum_{a_F \notin \bar{A}_F} \vec{\eta}(a_F) \cdot \vec{x}_F^{2,FT}({k^1_F}) \]

(A29)

where \( \vec{\eta}(a_F) \) is the vector of Lagrange multiplier on the final constraint under action \( a_F \). Following the same steps as in the proof of Proposition 1, this implies that for some \( \gamma_H > 0 \),

\[
\gamma_H \sum_{a_F \notin \bar{A}_F} \vec{\eta}(a_F) \cdot \vec{x}_F^{2,FT}({k^1_F}) = \vec{s}^1_{H,F} = \gamma_H \sum_{a_F \notin \bar{A}_F} \vec{\eta}(a_F) \cdot \vec{x}_F^{2,FT}({k^1_F}) \]  

(A30)

We now consider the planner’s first order condition with respect to second-period trade in any state \( \omega \) with Foreign action \( a_F(n) \notin \bar{A}_F \). Following same steps as in the proofs of Propositions 4 and 5, and letting \( \lambda_F^1, \lambda_F^2(m), \) and \( \kappa(m) \) be Lagrange multipliers on the
terms-of-trade and Foreign utility constraints, we obtain

\[
\mu(n)V_{H,x_g}^2(n) = \lambda^E \mu(n)V_{F,x_g}^2(n) + \mu(n)\lambda_F^2(n) \left[V_{F,x_g}^2(n) + x_F^2(n) \cdot V_{F,x_g}^2(n)\right] \\
+ \left[\lambda_F^1 x_F^1 \cdot V_{F,xk}^1 + \sum_{m=1}^M \mu(m)\lambda_F^2(m)\bar{x}^2_F(m) \cdot V_{F,xk}^2(m)\right] \cdot \tilde{k}_{F,x_g}^1(n) \\
- (\mu(n)\kappa(n) - \mu(n-1)\kappa(n-1))x_F^2(n) \cdot V_{F,x_g}^2(n) \\
+ \sum_{m=1}^{M-1} \mu(m)\kappa(m) \int_0^1 x_{F,m}^2(\zeta) \cdot V_{F,xk}^2 \cdot x_{F,m}^2(\zeta) d\zeta \cdot \tilde{k}_{F,x_g}^1(n) \\
- \eta_g(n) + \sum_{m \in \tilde{M}} \eta(m) \cdot \nu_{F,k_F^1} \cdot \tilde{k}_{F,x_g}^1(n)
\]  

(A31)

where \(\tilde{M}\) is the set of \(m\) for which \(a_F(m) \in \tilde{A}_F\). In the \(\beta \to 0\) limit, \(x_{F,k_F^1}^2(n) \to 0\) as firms have a vanishing incentive to invest on the basis of time-2 prices. We may therefore replace the final line of (A31) with \(-\eta_g(n) + o(\beta)\).

Multiplying both sides of (A31) by \(\gamma_H/(\gamma_F \mu(n)V_{F,x_g}^2)\) —where here \(\gamma_H\) and \(\gamma_F\) are the values associated with the solution of (A17)—and employing the definitions of elasticities from the main text and the proof of Propositions 4 and 5 implies that for any \(\omega\) with \(a_F(\omega) = a_F(n),\)

\[
1 + t_{H,g}^2(\omega) = 1 + t_{H,g}^2(\omega) - \frac{\gamma_H}{\mu(n)p_{F,g}^2(\omega)} \eta_g(n) + o(\beta) 
\]  

(A32)

Where here we have used the assumption that . Above, \(t_{H,g}^2(\omega)\) is given by

\[
\frac{\gamma_F}{\gamma_H} (1 + t_{H,g}^2(\omega)) = \lambda^E + \lambda_F^2(n)[1 + \Sigma_g^2(n)] - \left[\kappa(n) - \frac{\mu(n-1)}{\mu(n)}\kappa(n-1)\right] \Sigma_g^2(n) \\
+ \lambda_F^2(n) \Phi_g^2(n) + \sum_{m=1}^M \lambda_F^2(m)\Phi_g^2(m,n) + \sum_{m=1}^{M-1} \kappa(m)\Psi_g^2(m,n)
\]  

(A33)

We show in the proof of Propositions 4 and 5 that \(t_{H,g}^2(\omega)\) is Home’s unconstrained-optimal trade tax following action \(a_F(m)\).

Substituting (A33) into (A30), we have

\[
s_H^{-1}t_H^{-1} = E \left[1_{a_F(\omega) \notin \tilde{A}_F} \sum_{g \in G} p_{F,g}^2(\omega)t_{H,g}^2(\omega)x_{F,F,k_F^1}^2(\{k_F^1\})\right] + o(\beta^2) 
\]  

(A34)

where the fact that the last term is \(o(\beta^2)\) instead of \(o(\beta)\) follows from the observation that all \(p_{F,g}^2(\omega) \to 0\) as \(\beta \to 0\). This completes the proof.

---

\(^{28}\)We assume these multipliers exist.
A.6 Proof of Propositions 4 and 5

We begin from the primal formulation of the planner’s problem reached in the proof of Proposition 1, i.e. Equation A17. We study the “inner problem” of selecting \( x^*_F \), \( \{ \tilde{x}^2_F(a_F) \} \), and \( k^1_F \) given the optimal Home capital \( k^1_H \), Foreign geopolitical actions \( \{ a_F(\omega) \} \), and differences in Foreign economic welfare across geopolitical actions, i.e.

\[
\Delta V^2_F(m) \equiv V^2_F(\tilde{x}^2_F(a_F(m + 1), k^1_F)) - V^2_F(\tilde{x}^2_F(a_F(m), k^1_F)).
\]  

(A35)

Substituting in for \( k^1_p \) and ignoring terms that are held fixed in the inner problem, we rewrite this inner problem as

\[
\begin{align*}
\max_{\{ x_F, x_F^2(a_F) \}} & \quad V^1_H(-x^1_F, k^1_H^*) + \mathbb{E} \left[ V^2_H(-\tilde{x}^2_F(a_F(\omega)), k^1_H^*) \right] \\
& + \lambda^E V^1_F \left( x^1_F, k^1_F \left( x^1_F, \{ \tilde{x}^2_F(a_F(\omega)) \} \right) \right) + \lambda^E \mathbb{E} \left[ V^2_F \left( \tilde{x}^2_F(a_F(\omega)), k^1_F \left( x^1_F, \{ \tilde{x}^2_F(a_F(\omega)) \} \right) \right) \right] \\
\text{s.t.} & \quad V^1_{F,x} \left( x^1_F, k^1_F \left( x^1_F, \{ \tilde{x}^2_F(a_F(\omega)) \} \right) \right) \cdot x^1_F = 0 \\
& \quad V^2_{F,x} \left( \tilde{x}^2_F(a_F), k^1_F \left( x^1_F, \{ \tilde{x}^2_F(a_F(\omega)) \} \right) \right) \cdot \tilde{x}^2_F(a_F) = 0 \\
& \quad V^2_F \left( \tilde{x}^2_F(a_F(m + 1), k^1_F \left( x^1_F, \{ \tilde{x}^2_F(a_F(\omega)) \} \right) \right) \\
& \quad - V^2_F \left( \tilde{x}^2_F(a_F(m), k^1_F \left( x^1_F, \{ \tilde{x}^2_F(a_F(\omega)) \} \right) \right) = \Delta V^2_F(m)
\end{align*}
\]  

(A36)

Before taking the first-order conditions of this planning problem, we rewrite the Foreign utility constraints using the following observation: Consider any two trade vectors \( x^2_F \) and \( x^2_F' \) consistent with trade balance at capital \( k^1_F \), i.e.

\[
V^2_{F,x} \left( x^2_F, k^1_F \right) \cdot x^2_F = V^2_{F,x} \left( x^2_F', k^1_F \right) \cdot x^2_F' = 0.
\]  

(A37)

Letting \( x(\zeta) \) denote any smooth path between \( x^2_F \) and \( x^2_F' \), we have

\[
\begin{align*}
& \quad V^2_F \left( x^2_F', k^1_F \right) - V^2_F \left( x^2_F, k^1_F \right) \\
& = \int_0^1 V^2_{F,x} \left( x(\zeta), k^1_F \right) \cdot x'(\zeta) d\zeta \\
& = \int_0^1 \left[ -x(\zeta) \cdot V^2_{F,x} \left( x(\zeta), k^1_F \right) \cdot x'(\zeta) + \frac{d}{d\zeta} \left[ x(\zeta) \cdot V^2_{F,x} \left( x(\zeta), k^1_F \right) \right] \right] d\zeta \\
& = - \int_0^1 x(\zeta) \cdot V^2_{F,x} \left( x(\zeta), k^1_F \right) \cdot x'(\zeta) d\zeta + \left[ x(\zeta) \cdot V^2_{F,x} \left( x(\zeta), k^1_F \right) \right]_{\zeta=0}^{\zeta=1}
\end{align*}
\]  

(A38)

This implies that the last constraint in (A36) can be written as

\[
\int_0^1 x^2_F(m)(\zeta) \cdot V^2_{F,m} \left( x^2_F(m)(\zeta), k^1_F \left( x^1_F, \{ \tilde{x}^2_F(a_F(\omega)) \} \right) \right) \cdot x^2_F(m)'(\zeta) d\zeta = -\Delta V^2_F(m)
\]  

(A39)
We now consider the first-order conditions of this planning problem, letting $\lambda^1_F$ and $\mu(m)\lambda^2_F(m)$ denote the Lagrange multipliers on the terms-of-trade constraints and letting $\mu(m)\kappa(m)$ denote the Lagrange multipliers on the Foreign utility constraints.  

\[
V^1_{H,x_g} = \lambda^E V^1_{F,x_g} + \lambda^1_F \left[ V^1_{F,x_g} + x^1_F \cdot V^1_{F,xx_g} \right] \\
+ \left[ \lambda^1_F x^1_F \cdot V^1_{F,xx_k} + \sum_{m=1}^{M} \mu(m)\lambda^2_F(m) x^2_F(m) \cdot V^2_{F,xx_k}(m) \right] \cdot \tilde{k}^1_{F,x_g} \\
+ \sum_{m=1}^{M-1} \mu(m)\kappa(m) \int_0^1 x^2_{F,m}(\zeta) \cdot V^2_{F,xxk} \cdot x^2_{F,m}'(\zeta) d\zeta \cdot \tilde{k}^1_{F,x_g}(n) \\
= \mu(n)V^2_{H,x_g}(n) = \frac{\gamma H V^1_{H,x_g}}{\gamma F V^1_{F,x_g}} \\
\mu(n)\kappa(m) \int_0^1 x^2_{F,m}(\zeta) \cdot V^2_{F,xxk} \cdot x^2_{F,m}'(\zeta) d\zeta \cdot \tilde{k}^1_{F,x_g}(n)
\]

where above we have omitted some functions’ arguments where there is no loss of clarity, where—with a slight abuse of notation—we let $x^2_F(m) \equiv \tilde{x}^2_F(a_F(m))$ and $V^2_F(m) \equiv V^2_F(x^2_F(m), \tilde{k}^2_F)$, and where $\Delta \kappa(m) \equiv \kappa(m) - \kappa(m-1)$ and $\kappa(0) \equiv \kappa(M) \equiv 0$.

Next, recall the elasticities defined in the main text and further define

\[
\Phi^1_g = \sum_{d \in D} \frac{x^1_F \cdot \frac{\partial}{\partial x^1_F} \tilde{p}^1_F(x^1_F, k^1_F) \cdot \tilde{k}^1_{F,d}(x^1_F, \{x^2_F(\omega)\})}{\tilde{p}^2_{F,g}(x^1_F, k^1_F) \cdot \partial x^1_F(g)} \\
\Phi^2_g(n) = \sum_{d \in D} \frac{x^1_F \cdot \frac{\partial}{\partial x^1_F} \tilde{p}^1_F(x^1_F, k^1_F) \cdot \tilde{k}^1_{F,d}(x^1_F, \{x^2_F(\omega)\})}{\mu(n)\tilde{p}^2_{F,g}(x^2_F(n), k^1_F) \cdot \partial x^2_F(g)} \\
\Phi^2_g(m, n) = \sum_{d \in D} \frac{x^2_F(m) \cdot \frac{\partial}{\partial x^2_F} \left[ \frac{\mu(m)\tilde{p}^2_F(x^2_F(m), k^1_F) \cdot \tilde{k}^1_{F,d}(x^1_F, \{x^2_F(\omega)\})}{\mu(n)\tilde{p}^2_{F,g}(x^2_F(n), k^1_F)} \right]}{\mu(n)\tilde{p}^2_{F,g}(x^2_F(n), k^1_F) \cdot \partial x^2_F(g)} \\
\Psi^2_g(m, n) = \sum_{d \in D} \frac{x^2_F(m) \cdot \frac{\partial}{\partial x^2_F} \left[ \left[ \mu(m)\tilde{p}^2_F(x^2_F(m), k^1_F) \cdot \tilde{k}^1_{F,d}(x^1_F, \{x^2_F(\omega)\}) \right] \cdot \tilde{p}^2_{F,g}(x^2_F(n), k^1_F) \cdot \partial x^2_F(g) \right]}{\mu(n)\tilde{p}^2_{F,g}(x^2_F(n), k^1_F) \cdot \partial x^2_F(g)} 
\]

(A41)

With this notation, and recalling from Lemma 3 that

\[
1 + t^1_{H,g} = \frac{\gamma H V^1_{H,x_g}}{\gamma F V^1_{F,x_g}} \quad \text{and} \quad 1 + t^2_{H,g}(m) = \frac{\gamma H V^2_{H,x_g}(m)}{\gamma F V^2_{F,x_g}(m)} 
\]

(A42)

\[\text{We assume these multipliers exist.}\]
for some $\gamma_H, \gamma_F > 0$, we can rewrite (A40) as

$$
\frac{\gamma_F}{\gamma_H} (1 + t_{H,g}^1) = \lambda^E + \lambda_F^1 [1 + \Sigma_g^1]
+ \lambda_F^1 \Phi_g^1 + \sum_{m=1}^M \lambda_F^2(m) \Phi_g^1(m) + \sum_{m=1}^{M-1} \kappa(m) \Psi_g^1(m)
$$

$$
\frac{\gamma_F}{\gamma_H} (1 + t_{H,g}^2(n)) = \lambda^E + \lambda_F^2(n) [1 + \Sigma_g^2(n)] - \left[ \kappa(n) - \frac{\mu(n-1)}{\mu(n)} \kappa(n-1) \right] \Sigma_g^2(n)
+ \lambda_F^2(n) + \sum_{m=1}^M \lambda_F^2(m) \Phi_g^2(m, n) + \sum_{m=1}^{M-1} \kappa(m) \Psi_g^2(m, n)
$$

(A43)

Finally, we apply this general characterization to reach Propositions 4 and 5. First, to reach Proposition 4, consider the case where Foreign capital is fixed, so that $\Phi_g^1 = \Phi_g^1(m) = \Psi_g^1 = \Psi_g^2(m, n) = 0$. In this case, (A43) simplifies to

$$
\frac{\gamma_F}{\gamma_H} (1 + t_{H,g}^1) = \lambda^E + \lambda_F^1 [1 + \Sigma_g^1]
\frac{\gamma_F}{\gamma_H} (1 + t_{H,g}^2(n)) = \lambda^E + \lambda_F^2(n) [1 + \Sigma_g^2(n)] - \left[ \kappa(n) - \frac{\mu(n-1)}{\mu(n)} \kappa(n-1) \right] \Sigma_g^2(n)
$$

(A44)

as desired.

Second, to reach Proposition 5, let $x^{1\ast} > 0$ denote any bundle of goods (see the statement of the proposition) time-1 trade in the direction of which has no first-order impact on Foreign’s terms of trade or capital, i.e.

$$
\Sigma_g^1 \cdot x^{1\ast} = k_{F,x}^1 \cdot x^{1\ast} = 0.
$$

(A45)

By assumption, Home is indifferent to marginal transfers of $x^{1\ast}$ between countries at time 1, i.e. $V_{1,H}^E \cdot x^{1\ast} = \lambda^E V_{1,F}^E \cdot x^{1\ast}$. Putting together these observations and evaluating (A40) in the direction of $x^{1\ast}$ implies

$$
V_{1,H}^E \cdot x^{1\ast} = \lambda^E V_{1,F}^E \cdot x^{1\ast} + \lambda_F^1 V_{1,F}^E \cdot x^{1\ast} \implies \lambda_F^1 = 0
$$

(A46)

Finally, note that $\Phi_g^2(m, n), \Psi_g^2(m, n) \to 0$ as $\beta \to 0$ since, in the limit, time 2 does not affect the incentives of capital producers. Returning to (A43) with these observations, we obtain

$$
\frac{\gamma_F}{\gamma_H} (1 + t_{H,g}^1) = \lambda^E + \sum_{m=1}^M \lambda_F^2(m) \Phi_g^1(m) + \sum_{m=1}^{M-1} \kappa(m) \Psi_g^1(m)
\frac{\gamma_F}{\gamma_H} (1 + t_{H,g}^2(n)) = \lambda^E + \lambda_F^2(n) [1 + \Sigma_g^2(n)] - \left[ \kappa(n) - \frac{\mu(n-1)}{\mu(n)} \kappa(n-1) \right] \Sigma_g^2(n) + o(\beta)
$$

(A47)

This completes the proof.
\[ 1 + t^1_{H,g} = \frac{\gamma_H}{\gamma_G} \lambda^E \left[ 1 + \sum_{m=1}^{M} \frac{\lambda^2_f(m)}{\lambda^E} \Phi^1_g(m) + \sum_{m=1}^{M-1} \frac{\kappa(m)}{\lambda^E} \Psi^1_g(m) \right] \]
\[ 1 + t^2_{H,g}(n) = \frac{\gamma_H}{\gamma_G} \lambda^E \left[ 1 + \frac{\lambda^2_f(n)}{\lambda^E} \left[ 1 + \Sigma^2_g(n) \right] + o(\beta) \right] \]

(B) Proofs of the example

For ease of reading, we restate the assumptions of the example. On the household side, we suppose utility is quasi-linear. That is, for all times \( T \) and countries \( i \),

\[ u^T_i(c) = c_0 + \sum_{g \in G \setminus \{0\}} u^T_{ig}(c_g) \]

for some smooth sub-utility functions \( u^T_{ig} \) where \( \frac{\partial u^T_{ig}}{\partial c} > 0 \) and \( \frac{\partial^2 u^T_{ig}}{\partial c^2} < 0 \). We further focus on the limit where conflict is short-lived (i.e. \( \beta \to 0 \)).

On the production side, we suppose goods are organized into simple supply chains. Nonzero even-numbered goods \( g \) are “upstream goods” of which each country \( i \) has an exogenous endowment \( \bar{y}^T_{ig} \) at each time \( T \). Upstream goods \( g \) can be consumed directly at any time or converted one-for-one into capital of a type \( d_{g-1} \) at time 1. Odd-numbered goods \( g \) are “downstream goods” produced using capital of type \( d_g \) according to a production function \( f_{ig} \) in each country \( i \). That is,\(^{30}\)

\[ y^T_{ig} = \bar{y}^T_{ig} \quad \text{and} \quad k^T_{id_{g-1}} = \tau^1_g \quad \text{for } g \text{ even} \]
\[ y^T_{ig} = f_{ig}(k^T_{id_g}) \quad \text{for } g \text{ odd} \] \hfill (A49)

On the geopolitical side, we assume Foreign only has two actions: aggression or restraint which we denote \( a_A \) and \( a_R \) respectively. Home experiences a higher geopolitical utility when Foreign chooses restraint than when it chooses aggression, i.e. \( v_H(a_R) > v_H(a_A) \). We will denote by \( \rho(m) \) the endogenous weight that Home puts on Foreign’s economic utility in period 2 after action \( m \). This is defined \( 1 - \rho(m) = -\Delta \kappa(m) \) in the notation of Proposition 5.

Throughout, we assume that the second derivative of Home’s objective function in the second period is strictly negative.\(^{31}\)

\(^{30}\)Formally, the set of capital types is \( \mathcal{D} = \{d_{g-1} \mid g \in G, \ g \text{ non-zero and even} \} \) and the economy has goods and capital production possibilities frontiers given by

\[ G^T_i(y, k) = \max \left[ \max_{g \text{ even}} \left( y_g - \bar{y}^T_{ig} \right), \max_{g \text{ odd}} \left( y_g - f_{ig}(k_g) \right) \right] \]
\[ \Lambda^1_i(k^1, \{k^2(\omega)\}, \tau^1) = \max_{g \text{ even}} \left[ \max_{\omega \in \Omega} \left( k^1_{d_{g-1}} , \max_{\omega \in \Omega} k^2_{d_{g-1}}(\omega) \right) - \tau^1_g \right] \]

\(^{31}\)Formally, we assume that the second derivative of \( u^2_{H,g}(y^2_{H,g} + x_F) + \rho u^2_{F,g}(y^2_{F,g} - x_F) - (1 - \rho) \frac{\partial^2 u_{H,g}(y^2_{H,g} + x_F)}{\partial c^2} x_F \) with respect to \( x_F \) is strictly negative for all \( \rho \in [\rho(a_A), \rho(a_R)] \) where \( y^2_{ig} \) is the equilibrium level of production of country \( i \) in good \( g \).
Example. Suppose Home sanctions for aggression are severe enough that Home has lower economic utility when Foreign chooses aggression than when it chooses restraint. Then for any \( g \in \mathcal{G}\setminus\{0\} \), the direction of trade in \( g \) is consistent across aggression and restraint, and Home’s capital accumulation subsidies can then be characterized by the direction of trade of downstream (i.e., odd-numbered) goods. Specifically,

1. If Home imports good \( g \), then Home subsidizes productive capacity of good \( g \) domestically, i.e., \( s_{Hg}^1 > 0 \).
2. If Home exports good \( g \), then Home taxes productive capacity in good \( g \) domestically, i.e., \( s_{Hg}^1 < 0 \).

Proof. Because Home has lower economic utility when Foreign chooses aggression, Home will subsidize goods that have a higher price in the state of aggression than in restraint by Proposition 2. Prices at Home are simply \( p_{Hg}^2 = \frac{\partial u_{Hg}(y_{Hg}^2 + x_{Fg}^2)}{\partial c} \) which—given time-2 output \( y_{Hg}^2 \), which by assumption is the same in all states of the world—is decreasing in \( x_{Fg}^2 \). Therefore, Home will subsidize those goods for which Foreign net exports less of during aggression.

Next, we turn to finding what goods Foreign exports less of during conflict. By Proposition 5 and the fact that, with quasilinear preferences, there is no missing-markets motive, there exists a Lerner constant \( \gamma(m) > 0 \) for which second period policy satisfies

\[
p_{Hg}^2(m) = \gamma(m) \left( 1 + (1 - \rho(m))\Sigma_{Fg}^2(m) \right) p_{Fg}^2(m)
\]

in state \( m \) in the limit as \( \beta \to 0 \), where \( \Sigma_{Fg}^2(m) = -\left( \frac{\partial^2 u_{Fg}^2(y_{Fg}^2 - x_{Fg}^2(m))}{\partial c^2} \right) \left( \frac{\partial^2 u_{Fg}^2(y_{Fg}^2 - x_{Fg}^2(m))}{\partial c} \right) \).

WLOG, we may normalize the tax on the quasi-linear good to 0—so that \( \gamma(m) = 1 \)—and plug in for prices. Then foreign exports of good \( g \) satisfy

\[
\frac{\partial u_{Hg}^2(y_{Hg}^2 + x_{Fg}^2(m))}{\partial c} = \left( 1 + (1 - \rho(m))\Sigma_{Fg}^2(x_{Fg}^2(m)) \right) \frac{\partial u_{Fg}^2(y_{Fg}^2 - x_{Fg}^2(m))}{\partial c}
\]

where \( \Sigma_{Fg}^2(x) = -\left( \frac{\partial^2 u_{Fg}^2(y_{Fg}^2 - x)}{\partial c^2} \right) \left( \frac{\partial^2 u_{Fg}^2(y_{Fg}^2 - x)}{\partial c} \right) \) is the functional form of the foreign terms-of-trade elasticity.

We know that Home wants to reward Foreign in restraint and punish Foreign in aggression. Therefore, \( \rho(a_R) \geq \rho(a_A) \). Thus, we need to compare implied exports of good \( g \) under those two different \( \rho \). We do that by defining foreign exports as a function of \( \rho \), \( \tilde{x}_{Fg}^2(\rho) \), that satisfies the first order necessary condition from (A50), i.e.

\[
\frac{\partial u_{Hg}^2(y_{Hg}^2 + \tilde{x}_{Fg}^2(\rho))}{\partial c} = \left( 1 + (1 - \rho)\Sigma_{Fg}^2(\tilde{x}_{Fg}^2(\rho)) \right) \frac{\partial u_{Fg}^2(y_{Fg}^2 - \tilde{x}_{Fg}^2(\rho))}{\partial c}
\]

Given our assumption that the second derivative of \( u_{Hg}^2(y_{Hg}^2 + x_F) + \rho u_{Fg}^2(y_{Fg}^2 - x_F) - (1 - \rho a_R)
\]

\[32\text{This follows from the fact that Foreign’s period 2 utility is increasing in the weight Home puts on Foreign, }\rho(a).\text{ If }\rho(a_R) < \rho(a_A),\text{ then Foreign’s utility would be higher in the aggression state. Home could then increase its utility by choosing the trade vector that gives it the highest economic utility of those implied by }\rho_F(a_R)\text{ and }\rho_F(a_A).\text{ This would increase Home’s economic welfare and decrease Foreign’s probability of taking aggression.}
\( \rho \frac{\partial u_{Fg}(y^2_{Fg} - x_F)}{\partial c} x_F \) is strictly negative for all \( \rho \in [\rho(a_A), \rho(a_R)] \), the implicit function theorem implies that the function is locally defined and differentiable. In particular,

\[
\frac{\partial^2 x^2_{Fg}(\rho)}{\partial \rho} = -\bar{x}^2_{Fg}(\rho) \frac{1}{1 + t^H_g(\rho)} \frac{\Sigma^2_{Fg}(\bar{x}^2_{Fg}(\rho))}{\Omega_g(\bar{x}^2_{Fg}(\rho))}
\]  

(A52)

where \( \Sigma^2_{Hg}(x) = -\left( \frac{\partial^2 u^2_{Hg}(y^2_{Hg} - x)}{\partial c^2} \right) / \left( \frac{\partial u^2_{Hg}(y^2_{Hg} - x)}{\partial c} \right) \) and

\[
\Omega_g(x) = \frac{\Sigma^2_{Hg}(-x) - \Sigma^2_{Fg}(x)}{1 + (1 - \rho) \Sigma^2_{Fg}(x) \Sigma^2_{Fg}(x)} x \frac{\partial^2 \Sigma^2_{Fg}(x)}{\partial x}
\]

Evaluated at \( \bar{x}^2_{Fg}(\rho) \),

\[
\Omega_g(\bar{x}^2_{Fg}(\rho)) = \bar{x}^2_{Fg}(\rho) \frac{\partial}{\partial x_F} \left\{ \log \left( \frac{\partial [u^2_{Hg}(y^2_{Hg} + x_F)]}{\partial x_F} \right) \right\} + \log \left( \frac{\partial [\lambda u^2_{Fg}(y^2_{Fg} - x_F) - (1 - \rho) \frac{\partial u^2_{Fg}(y^2_{Fg} - x_F)}{\partial c} x_F]}{\partial x_F} \right) \right\} \bigg|_{x_F = \bar{x}^2_{Fg}(\rho)}
\]

\[
\frac{\partial^2 x^2_{Fg}(\rho)}{\partial \rho} = \bar{x}^2_{Fg}(\rho) \left\{ \frac{\partial^2 [u^2_{Hg}(y^2_{Hg} + x_F)]}{\partial x_F^2} + \frac{\partial^2 [\lambda u^2_{Fg}(y^2_{Fg} - x_F) - (1 - \rho) \frac{\partial u^2_{Fg}(y^2_{Fg} - x_F)}{\partial c} x_F]}{\partial x_F^2} \right\} \bigg|_{x_F = \bar{x}^2_{Fg}(\rho)}
\]

\[
= \frac{\partial^2 [u^2_{Hg}(y^2_{Hg} + x_F)]}{\partial x_F^2} + \rho \frac{\partial^2 [\lambda u^2_{Fg}(y^2_{Fg} - x_F) - (1 - \rho) \frac{\partial u^2_{Fg}(y^2_{Fg} - x_F)}{\partial c} x_F]}{\partial x_F^2} \bigg|_{x_F = \bar{x}^2_{Fg}(\rho)}
\]

The last equality follows because it is evaluated at \( \bar{x}^2_{Fg}(\rho) \) which satisfies equation (A51). By our assumption on the second derivative, \( \Omega_g \) then has the opposite sign of \( \bar{x}^2_{Fg}(\rho) \). Since \( \Sigma^2_{Fg}(\bar{x}^2_{Fg}(\rho)) \) has the same sign as \( \bar{x}^2_{Fg}(\rho) \) and \( 1 + t^H_g(\rho) \) is always positive by assumption, (A52) therefore implies \( \frac{\partial x^2_{Fg}(\rho)}{\partial \rho} \) has the same sign as \( \bar{x}^2_{Fg}(\rho) \). So as \( \rho \) increases, Foreign exports more of the goods it exports and imports more of the goods it imports.

Since \( \rho(a_R) \geq \rho(a_A) \), it follows that in periods of aggression, Home has less of the goods that Foreign exports—and so has higher prices for such goods, as discussed above. Therefore, Home should subsidize production capacity of those goods. Similarly, Home will have more of the goods that Foreign imports during aggression. Therefore, Home should tax productive capacity of those goods.

\[ \square \]

**Example.** Suppose that time-2 Home trade policy is constrained to free trade if and only if Foreign chooses restraint, and that Home’s unconstrained policy would reward Foreign for restraint more than does free trade. Then Home’s capital subsidies can be characterized by the direction of trade of downstream (i.e. odd-numbered) goods under free trade. Specifically,

- If Home imports good \( g \), then Home taxes productive capacity of good \( g \) domestically,
If Home exports good $g$, then Home subsidizes productive capacity of good $g$ domestically, i.e. $s_{d_g}^1 > 0$.

**Proof.** Tariffs are restricted to zero in and only in the case of restraint. Therefore, Home’s capital subsidies for productive capacity of good $g$ is

$$s_{H,d}^1 t_{H,d}^1 = t_{H}^{2*}(a_R) p_{F}^2(a_R) \cdot \frac{d \tau_{H}^{2*}}{dk_{H,d}^2}$$

by Proposition 3.

In order to sign Home’s capital subsidies, we now proceed to sign the terms of (A53). First, we argue that $t_{H}^{2*}(a_R)$ has the same sign as the direction of Home’s net exports. By Proposition 5,

$$t_{H}^{2*}(a_R) = (1 - \rho(a_R)) \Sigma_{Fg}^2(a_R)$$

when the tax on the quasi-linear good is normalized to 0. Since we have assumed that Home would like to reward Foreign for restraint more than does free trade, $\rho(a_R) > 1$. Then Home’s desired taxes have the opposite sign as $\Sigma_{Fg}^2(a_R)$, which is positive for imports and negative for exports. So, if Foreign exports a good, Home’s desired trade tax is positive (an export subsidy). Conversely, if Foreign imports a good, Home’s desired tax is negative (an import subsidy).

Second, we characterize $\frac{d x_{H}^{2*}}{d k_{H,d}^1}$. In free trade, exports of a downstream good $g$ must satisfy

$$\frac{\partial u_{H}^2(g_{H}(k_{1}^{H_{d_g}}) - x_{H}^{2*})}{\partial c} = \frac{\partial u_{F}^2(g_{F}^{2} + x_{H}^{2*})}{\partial c}.$$ 

Implicitly differentiating

$$\frac{\partial^2 u_{H}^2(g_{H}(k_{1}^{H_{d_g}}) - x_{H}^{2*})}{\partial c^2} \left[ f_{H}(k_{1}^{H_{d_g}}) - \frac{d x_{H}^{2*}}{d k_{H,d}^1} \right] = \frac{\partial^2 u_{F}^2(g_{F}^{2} + x_{H}^{2*})}{\partial c^2} \frac{d x_{H}^{2*}}{d k_{H,d}^1}.$$ 

Rearranging, we find

$$\frac{d x_{H}^{2*}}{d k_{H,d}^1} = \frac{\frac{\partial^2 u_{H}^2(g_{H}(k_{1}^{H_{d_g}}) - x_{H}^{2*})}{\partial c^2}}{\frac{\partial^2 u_{H}^2(g_{H}(k_{1}^{H_{d_g}}) - x_{H}^{2*})}{\partial c^2} + \frac{\partial^2 u_{F}^2(g_{F}^{2} + x_{H}^{2*})}{\partial c^2}} f_{H}(k_{1}^{H_{d_g}}) > 0,$$

and $d x_{H}^{2*}/d k_{H,d}^1 = 0$ for $g' \neq g, 0$. Combining these observations, we conclude that Home subsidizes productive capacity of good $g$ if Home exports the good in free trade and taxes productive capacity if Home imports the good.

**Example.** Home would like to discourage Foreign investment in productive capacity of a downstream (i.e. odd-numbered) good $g$ if Foreign’s consumption utility for $g$ exhibits prudence, i.e. $\frac{\partial^2 u_{F}^2(g_{F}^{2})}{\partial c^2} > 0$. By contrast, if Foreign’s consumption utility for $g$ exhibits anti-
prudence, i.e. $\frac{\partial^3 u_{F,g}}{\partial \zeta^3} < 0$, Home would like to encourage Foreign investment in productive capacity. And if Foreign’s prudence is 0, Home does not want to change Foreign’s investment.

**Proof.** As discussed in the main text, Home’s desire to discourage investment in productive capacity of good $g$ is captured by the following term:

$$X_g \equiv \frac{\partial}{\partial k_{F,g}} \left[ \int_0^1 \tilde{x}^2_F(\zeta) \cdot \frac{\partial}{\partial x_F} \left[ \tilde{p}_F(\tilde{x}^2_F(\zeta), k_{F,g}) \right] \cdot \tilde{x}^2_F(\zeta) d\zeta \right]$$

where $\tilde{x}_{F,m}(\zeta)$ is a smooth path from $x^*_F(a_R)$ and $x^*_F(a_A)$. Normalizing the price of the quasi-linear good to 1, prices in Foreign of good $g'$ are

$$p^2_{F,g'} = \frac{\partial u^2_{F,g}(f_{F,g}(k_{F,d,g}) - x_{F,g'})}{\partial \zeta}.$$ 

Therefore,

$$\frac{\partial}{\partial k_{F,d,g}} \left[ \tilde{x}^2_F(\zeta) \cdot \frac{\partial}{\partial x_F} \left[ \tilde{p}_F(\tilde{x}^2_F(\zeta), k_{F,g}) \right] \cdot \tilde{x}^2_F(\zeta) \right] = -\tilde{x}^2_F(\zeta) \frac{\partial^3 u^2_{F,g}(\zeta)}{\partial \zeta^3} f'_{F,g}(k_{F,d,g}) \tilde{x}^2_F(\zeta) \tilde{x}^2_F(\zeta)$$

We next turn to finding the trade vectors in the second period. By Proposition 5, second period trade policy must be such that

$$p^2_{H,g}(m) = (1 + (1 - \rho(m))\Sigma^2_{F,g}(m)) p^2_{F,g}(m)$$

in state $m$ as $\beta \to 0$ when the tax on the quasi-linear good is normalized to 0. Normalizing the tax on the quasi-linear good to 1, we can then get an expression for $x^2_{F,g}$ as a function of $\rho$,

$$\frac{\partial u^2_{H,g}(y^2_{H,g} + x^2_{F,g}(\rho))}{\partial \zeta} = \left(1 + (1 - \rho)\Sigma^2_{F,g}(x_{F,g}(\rho)) \right) \frac{\partial u^2_{F,g}(y^2_{F,g} - x^2_{F,g}(\rho))}{\partial \zeta}$$

Then by our assumption that the second derivative of $u^2_{H,g}(y^2_{H,g} + x) + \rho u^2_{F,g}(y^2_{F,g} - x) - (1 - \rho)\frac{\partial u^2_{F,g}(y^2_{F,g} - x)}{\partial \zeta} x$ is strictly negative for all $\rho \in [\rho(a_A), \lambda(a_R)]$, the implicit function theorem implies that the function is locally defined and differentiable. In particular,

$$\frac{\partial \tilde{x}^2_{F,g}(\rho)}{\partial \lambda} = -\tilde{x}^2_{F,g}(\rho) \frac{1}{1 + t^H_{F,g}(\rho)} \Sigma^2_{F,g}(\tilde{x}^2_{F,g}(\rho))$$

where

$$\Omega_g(x) = \Sigma^2_{H,g}(-x) - \Sigma^2_{F,g}(x) - \frac{(1 - \lambda)\Sigma^2_{F,g}(x)}{1 + (1 - \lambda)\Sigma^2_{F,g}(x)} \frac{x}{\Sigma^2_{F,g}(x)} \frac{\partial \Sigma^2_{F,g}(x)}{\partial x}.$$ 

We can then use the path induced by $\rho$ from $\rho(a_A)$ to $\rho(a_R)$ for the path of the integral that
defines $X_g$. That is,

$$X_g = - \int_{\rho(\alpha_\Lambda)}^{\rho(\alpha_R)} \tilde{x}_{Fg}^2(\rho) \frac{\partial^3 u_{Fg}^2(\rho)}{\partial c^3} f'_{Fg}(k_{F,dg}) \xi_{Fg}'(\rho) d\rho$$

$$= \int_{\rho(\alpha_\Lambda)}^{\rho(\alpha_R)} \tilde{x}_{Fg}^2(\rho) \frac{\partial^3 u_{Fg}^2(\rho)}{\partial c^3} f'_{Fg}(k_{F,dg}) \frac{1}{1 + h^H(\rho) \Omega_g(\tilde{x}_{Fg}^2(\rho))} d\rho.$$ 

To sign this, note that at $\tilde{x}_{Fg}^2(\rho)$,

$$\Omega_g(\tilde{x}_{Fg}^2(\rho)) = \frac{\partial^2 u_{Hg}^2(y_{Hg}^2 + x_F)}{\partial x_F} \left\{ \log \left( \frac{\partial [u_{Hg}^2(y_{Hg}^2 + x_F)]}{\partial x_F} \right) + \log \left( \frac{\partial \left[ \lambda u_{Fg}^2(y_{Fg}^2 - x_F) - (1 - \rho) \frac{\partial u_{Fg}^2(y_{Fg}^2 - x_F)}{\partial c} x_F \right]}{\partial x_F} \right) \right\} | \tilde{x}_F = \tilde{x}_{Fg}^2(\rho)$$

$$= \frac{\partial^2 u_{Hg}^2(y_{Hg}^2 + x_F)}{\partial x_F} \frac{\partial^2 [u_{Hg}^2(y_{Hg}^2 + x_F)]}{\partial x_F} + \frac{\partial \left[ \lambda u_{Fg}^2(y_{Fg}^2 - x_F) - (1 - \rho) \frac{\partial u_{Fg}^2(y_{Fg}^2 - x_F)}{\partial c} x_F \right]}{\partial x_F} \right\} | \tilde{x}_F = \tilde{x}_{Fg}^2(\rho)$$

The last equality follows because it is evaluated at $\tilde{x}_{Fg}^2(\rho)$ which satisfies equation (A51). By our assumption on the second derivative, $\Omega_g$ then has the opposite sign of $\tilde{x}_{Fg}^2(\rho)$. On the other hand, $\Sigma_{Fg}^2$ has the same sign as $\tilde{x}_{Fg}^2(\rho)$. Therefore, the integral $X_g$ is negative if $\frac{\partial^3 u_{Fg}^2(\rho)}{\partial c^3} > 0$. That is, Home wants to discourage Foreign investment in capital for good $g$. Furthermore, the integral $X_g$ is positive if $\frac{\partial^3 u_{Fg}^2(\rho)}{\partial c^3} < 0$ and 0 if the third derivative is 0. \( \square \)

**Example.** Suppose that Home has weak preferences over Foreign’s geopolitical actions and that Home is close to indifferent between consumption at Home and Foreign under both aggression and restraint. All else equal,\(^{33}\) Home would like to discourage a unit of capital investment in the productive capacity of good $g$ more than of good $g'$ if

1. Good $g$ has a higher relative prudence than good $g'$, i.e. $\xi_{Fg} > \xi_{Fg'}$. 
2. Foreign’s trade openness to good $g$ is higher, i.e. $|x_{Fg}/c_{Fg}| > |x_{Fg'}/c_{Fg'}|$.

**Proof.** We start with the expression we found in proving the previous example,

$$X_g = \int_{\rho(\alpha_\Lambda)}^{\rho(\alpha_R)} \tilde{x}_{Fg}^2(\rho) \frac{\partial^3 u_{Fg}^2(\rho)}{\partial c^3} f'_{Fg}(k_{F,dg}) \frac{1}{1 + h^H(\rho) \Omega_g(\tilde{x}_{Fg}^2(\rho))} d\rho.$$ 

\(^{33}\)Home’s desire to change Foreign’s productive capacity for good $g$ depends on Foreign trade openness $x_{Fg}/c_{Fg}$, the coefficient of relative risk aversion in both countries $\sigma_{ig} = -c_{ig} \frac{\partial u_{ig}^2}{\partial c_{ig}}$, consumption of Home relative to Foreign $c_{Fg}/c_{Hg}$, Foreign’s relative prudence $\xi_{Fg} = -c_{Fg} \frac{\partial u_{Fg}^2}{\partial c_{Fg}}$, and the marginal product of Foreign capital $p_{Fg}^2 / k_{Fg}$. 

53
We then define relative prudence,

\[ \xi_{ig}(\rho) \equiv -\frac{c_{ig}^2(\rho)}{\xi_{ig}(\rho)} \frac{\partial^2 u_{ig}(\rho)}{\partial c^2}, \]

and the coefficient of relative risk aversion,

\[ \sigma^2_{ig}(\rho) \equiv -\frac{c_{ig}^2(\rho)}{\xi_{ig}(\rho)} \frac{\partial^2 u_{ig}(\rho)}{\partial c}. \]

We can then rewrite

\[ X_g = \int_{\rho(a_A)}^{\rho(a_R)} p_{Fg}^2(\rho) \left( x_{Fg}(\rho) \sigma_{Fg}(\rho) \right)^2 \frac{1}{1 + \frac{\partial H(\rho)}{\partial \rho}} \frac{f'_{Fg}(k_{F,d_g}^1)}{\Omega(\rho)} (\rho(a_R) - \rho(a_A)) \]

Linearizing around the point of free trade with \( \rho = 1 \), we find

\[ X_g \approx p_{Fg}^2(1) \left( \frac{x_{Fg}(1)}{c_{Fg}(1)} \right)^2 \sigma_{Fg}(1) \xi_{Fg}(1) \frac{f'_{Fg}(k_{F,d_g}^1)(\rho(a_R) - \rho(a_A))}{1 + \frac{\partial H(\rho)}{\partial \rho}} \]

\[ = -p_{Fg}^2(1) \left( \frac{x_{Fg}(1)}{c_{Fg}(1)} \right)^2 \sigma_{Fg}(1) \xi_{Fg}(1) \frac{f'_{Fg}(k_{F,d_g}^1)(\rho(a_R) - \rho(a_A))}{1 + \frac{\partial H(\rho)}{\partial \rho}}. \]

We then immediately get the comparative statics from this expression changing \( \xi_{Fg}(1) \) and \( \frac{x_{Fg}^2(1)}{c_{Fg}^2(1)} \) holding everything else fixed.

**Example.** Foreign capital investment in the capacity to produce any downstream good \( g \) is determined by its peacetime exports of that good, \( x_{Fg}^1 \), and the corresponding upstream intermediate good \( g + 1 \), \( x_{F,g+1}^1 \). Furthermore,

\[ \frac{d\tilde{k}_{F,d_g}^1}{dx_{Fg}^1} > 0 \quad \text{and} \quad \frac{d\tilde{k}_{F,d_g}^1}{dx_{F,g+1}^1} < 0. \]

**Proof.** The Foreign producer of capital \( d_g \) solves the problem

\[ \max_{k_{d_g}^1, \tau_{d_g}^1} \left( r_{d_g}^1 + r_{d_g}^2(a_R) + r_{d_g}^2(a_A) \right) k_{d_g}^1 - p_{g+1}^1 \tau_{d_g}^1 \]

such that

\[ \tilde{k}_{d_g}^2 = \tau_{d_g}^1. \]

It follows that

\[ r_{d_g}^1 + r_{d_g}^2(a_R) + r_{d_g}^2(a_A) = p_{g+1}^1. \]
Normalizing the price of the quasi-linear good to 1, these prices are defined by

\[ r^1_{d_g} = \frac{\partial u^1_{F_g}(f_{F_g}(k^1_{d_g}) - x^1_{F_g})}{\partial c} f'_{F_g}(k^1_{d_g}) \]

\[ r^2_{d_g}(m) = \beta \mu(m) \frac{\partial u^2_{F_g}(f_{F_g}(k^1_{d_g}) - x^2_{F_g})}{\partial c} f'_{F_g}(k^1_{d_g}) \]

\[ p^1_{g+1} = \frac{\partial u^1_{F,g+1}(\bar{y}^1_{F,g+1} - x^1_{F,g+1})}{\partial c}, \]

where \( \mu(m) \) is the probability that state \( m \) occurs. Taking the limit as \( \beta \to 0 \), we find that

\[ \frac{\partial u^1_{F_g}(f_{F_g}(k^1_{d_g}) - x^1_{F_g})}{\partial c} f'_{F_g}(k^1_{d_g}) = \frac{\partial u^1_{F,g+1}(\bar{y}^1_{F,g+1} - x^1_{F,g+1})}{\partial c}, \]

implicitly defines \( k^1_{d_g} \) as a function of \( x^1_{F,g} \) and \( x^1_{F,g+1} \). Totally differentiating,

\[ 0 = \frac{\partial^2 u^1_{F_g}(f_{F_g}(k^1_{d_g}) - x^1_{F_g})}{\partial c^2} f''_{F_g}(k^1_{d_g}) \left[ f'_{F_g}(k^1_{d_g}) dk^1_{d_g} - dx^1_{F_g} \right] \]

\[ + \frac{\partial u^1_{F_g}(f_{F_g}(k^1_{d_g}) - x^1_{F_g})}{\partial c} f''_{F_g}(k^1_{d_g}) dk^1_{d_g} \]

\[ - \frac{\partial^2 u_{F,g+1}(\bar{y}^1_{F,g+1} - x^1_{F,g+1})}{\partial c^2} dx^1_{F,g+1} \cdot \]

Rearranging, we find

\[ dk^1_{d_g} = \frac{\partial^2 u^1_{F_g}(f_{F_g}(k^1_{d_g}) - x^1_{F_g})}{\partial c^2} f''_{F_g}(k^1_{d_g}) dx^1_{F,g} - \frac{\partial^2 u^1_{F,g+1}(\bar{y}^1_{F,g+1} - x^1_{F,g+1})}{\partial c^2} dx^1_{F,g+1}. \]

Since \( u^1_{F_g}, u^1_{F,g+1} \), and \( f_{F_g} \) are all increasing and concave, \( \frac{dk^1_{d_g}}{dx^1_{F_g}} > 0 \) and \( \frac{dk^1_{d_g}}{dx^1_{F,g+1}} < 0. \)